Price Dispersion in the New Keynesian Model

Damian Romero*

July 2020

In the standard New Keynesian model we typically get the following object

$$d_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di, \tag{1}$$

which corresponds to the price dispersion of firm i with respect to the aggregate price level. Note that this also applies in other kinds of models, such as multisector ones.
The first goal is to show that, up to a first order log-linear approximation, this term equals to one, and it is not relevant for the equilibrium determination in log-linearized models[1]. The second goal of this note is to show the specific recursive form of this price distortion when the nonlinear solution method is taken into account.[2]

Log-linear approximation

We show first that $p_t \simeq \int p_{it} di$, where lower case letters corresponds to the logarithm version of the variable. We start by using the definition of the aggregate CPI.

$$\begin{aligned} P_t &= \left(\int_0^1 P_{it}^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}} \\ 1 &= \left(\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}. \end{aligned}$$

Then we get

$$1 = \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{1-\varepsilon} di = \int_0^1 \exp[(1-\varepsilon)(p_{it}-p_t)] di.$$

Taking a second order approximation around $p_{it} = p_t$

*Email: dromeroc@bcentral.cl

[1]This is no longer true when the model is approximated around a trend inflation (i.e, different to zero steady state inflation), or when the model is solved in levels.

[2]Note that the price dispersion will be an endogenous state of the model under these circumstances.

$$
\begin{aligned}
1 &\simeq \int_0^1 [e^0 + e^0(1-\varepsilon)(p_{it} - p_t) + \frac{1}{2}e^0(1-\varepsilon)^2(p_{it} - p_t)^2]di \\
&\simeq 1 - (1-\varepsilon)p_t + (1-\varepsilon)\int_0^1 p_{it}di + \frac{(1-\varepsilon)^2}{2}\int_0^1 (p_{it} - p_t)^2 di \\
p_t &\simeq \underbrace{\int_0^1 p_{it}di}_{\text{first order term}} + \underbrace{\frac{1-\varepsilon}{2}\int_0^1 (p_{it} - p_t)^2 di,}_{\text{second order term}}
\end{aligned}
$$

which completes the first part.
Then, we show the main result by taking a second order approximation

$$
\begin{aligned}
\int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di &= \int_0^1 \exp\left[-\left(\frac{\varepsilon}{1-\alpha}\right)(p_{it} - pt)\right] di \\
&\simeq \int_0^1 \left[e^0 - e^0\left(\frac{\varepsilon}{1-\alpha}\right)(p_{it} - pt) + \frac{1}{2}\left(\frac{\varepsilon}{1-\alpha}\right)^2 (p_{it} - pt)^2\right] di \\
&= 1 + \left(\frac{\varepsilon}{1-\alpha}\right)p_t - \left(\frac{\varepsilon}{1-\alpha}\right)\int_0^1 p_{it}di + \frac{1}{2}\left(\frac{\varepsilon}{1-\alpha}\right)^2 \int_0^1 (p_{it} - p_t)^2 di.
\end{aligned}
$$

Replacing our first result for p_t, the right-hand side reads as

$$
\begin{aligned}
&= 1 + \left(\frac{\varepsilon}{1-\alpha}\right)\left(\int_0^1 p_{it}di + \frac{1-\varepsilon}{2}\int_0^1 (p_{it} - p_t)^2 di\right) - \left(\frac{\varepsilon}{1-\alpha}\right)\int_0^1 p_{it}di + \frac{1}{2}\left(\frac{\varepsilon}{1-\alpha}\right)^2 \int_0^1 (p_{it} - p_t)^2 di \\
&= 1 + \left(\frac{1-\varepsilon}{2}\right)\int_0^1 (p_{it} - p_t)^2 di + \frac{1}{2}\left(\frac{\varepsilon}{1-\alpha}\right)^2 \int_0^1 (p_{it} - p_t)^2 di \\
&= 1 + \underbrace{\frac{\varepsilon(1-\alpha+\alpha\varepsilon)}{(1-\alpha)^2}\int_0^1 (p_{it} - p_t)^2 di}_{\text{second order term}} > 1,
\end{aligned}
$$

where the last result holds because $\alpha \in (0, 1)$.
Therefore,

$$
d_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di \simeq 1,
$$

which concludes the proof.

Recursive form

First, note that (1) can de decomposed as

$$
d_t = \int_0^\theta \left(\frac{P_{it}}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di + \int_\theta^1 \left(\frac{P_{it}}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di,
$$

where θ is the fraction of firms that cannot reset its price in a given period. Using the Calvo property of the model, the previous expression can be written as

$$
\begin{aligned}
d_t &= \int_0^\theta \left(\frac{P_{it-1}}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di + \int_\theta^1 \left(\frac{P_t^*}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di \\
&= \int_0^\theta \left(\frac{P_{it-1}}{P_{t-1}}\right)^{-\frac{\varepsilon}{1-\alpha}} \left(\frac{P_{t-1}}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} di + (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} \\
&= \left(\frac{P_{t-1}}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} \theta \int_0^1 \left(\frac{P_{it-1}}{P_{t-1}}\right)^{-\frac{\varepsilon}{1-\alpha}} di + (1-\theta)\left(\frac{P_t^*}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}},
\end{aligned}
$$

where P_t^* is the optimal reset price, which is common across firms under the assumption that they face the same marginal cost.

Defining $p_t^* = P_t^*/P_t$ and $\Pi_t = P_t/P_{t-1}$, the previous expression can be written as

$$
d_t = \theta \Pi_t^{\frac{\varepsilon}{1-\alpha}} d_{t-1} + (1-\theta)(p_t^*)^{-\frac{\varepsilon}{1-\alpha}},
$$

which denotes the recursive form of price dispersion.[3]

[3] The important thing about this result is that we do not need to keep track of every individual price charged by monopolistic firms. For this end it is enough to know the evolution of the price dispersion.