Online Appendix: Inequality, Nominal Rigidities, and Aggregate Demand

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A Model's Details

A.1 Baseline Model

Households. The economy is populated by a continuum of households of mass 1, where a fraction λ cannot borrow, lend or own firms, while the remainder $1-\lambda$ has full access to financial markets and owns the firms in the economy. We refer to the former as constrained agents, denoted by c, and to the latter as unconstrained, denoted by u. Each household is composed by a continuum of members that supply differentiated labor varieties denoted by $j \in [0,1]$. We assume there is perfect insurance within the household, which equalizes members' consumption.

Households' lifetime utility is given by

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \chi_{t+k} \left(\frac{(C_{t+k}^K)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{N_{t+k}^K(j)^{1+\varphi}}{1+\varphi} dj \right), \tag{A.1}$$

for $K \in \{u, c\}$, where χ_t represents a shock to preferences, C_t^K is final good consumption and $N_t^K(j)$ denotes hours worked supplied to variety j.

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Households face the following period resource constraint

$$P_t C_t^K + Q_t B_t^K = B_{t-1}^K + \int_0^1 W_t(j) N_t^K(j) dj + D_t^K. \tag{A.2}$$

Earnings are given by labor income $\int_0^1 W_t(j) N_t^K(j) dj$ and profits D_t^K, which proceed from firms ownership. $P_t C_t^K$ is total (nominal) expenditure on the final good and $Q_t B_t^K$ are bond purchases, where P_t is the price of the final good while Q_t is the price of bonds.

We assume the preference shock follows an exogenous AR(1) process, given by

$$\log \chi_t = (1 - \rho_\chi) \log \chi + \rho_\chi \log \chi_{t-1} + \sigma_\chi \varepsilon_t^\chi.$$

Intertemporal optimization implies the following Euler equation for unconstrained households

$$1 = R_t \mathbb{E}_t \left\{ \beta \frac{\chi_{t+1}}{\chi_t} \left(\frac{C_t^u}{C_{t+1}^u} \right)^\sigma \frac{1}{\Pi_{p,t+1}} \right\}, \tag{A.3}$$

where $R_t = 1/Q_t$ and $\Pi_{p,t+1} = P_{t+1}/P_t$. Constrained households have no access to financial markets. Hence, their consumption equals current income from labor

$$C_t^c = \frac{W_t}{P_t} N_t. \tag{A.4}$$

Finally, aggregate consumption is given by

$$C_t = (1 - \lambda) C_t^u + \lambda C_t^c. \tag{A.5}$$

Final Good Producers. Firms producing the final good operate in a perfectly competitive environment and combine a continuum of measure one of intermediate goods $Y_t(i)$ to produce a homogeneous final good Y_t according to

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_p - 1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}, \tag{A.6}$$

where ϵ_p is the elasticity of substitution among good varieties.

Solving the optimization problem of the firm, we obtain the following demand function for intermediate inputs

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} Y_t, \tag{A.7}$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon_p} di\right)^{\frac{1}{1-\epsilon_p}}$ is the price of the final good.

Intermediate Goods Producers. There is a continuum of intermediate firms, indexed by $i \in [0, 1]$. These firms operate in a monopolistic competitive environment. Hence, each firm produces a single-differentiated good and operates as a monopoly in its own market. Intermediates production technology is given by

$$Y_t(i) = N_t(i)^{1-\alpha}, \tag{A.8}$$

where $Y_t(i)$ is firm i output and $N_t(i)$ is labor input. We assume that each firm i demands different kinds of labor provided by the households, with an elasticity of substitution ϵ_w. Thus, we have $N_t(i) = \left(\int_0^1 N_t(i,j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$, where $N_t(i,j)$ is the amount of labor variety j demanded by firm i. Then, a standard cost minimization problem derives the demand for each labor variety

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t(i), \tag{A.9}$$

where $W_t(j)$ is the wage of labor variety j. From Equation (A.9), note that the total demand across firms for variety j is given by $N_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t$.

Firms also face price rigidities. In the next sections we consider two different types of rigidities: prices set in advance and Calvo pricing, which we will describe in detail later. Assuming prices are set in advance allows us to study the role of nominal rigidities in aggregate demand analytically. On the other hand, with Calvo pricing, we take into account the role of expectations and dynamics to describe the impact of nominal rigidities. We provide further details below.

Monetary Policy. We assume that the monetary authority follows a Taylor rule that is subject to the zero lower bound, given by

$$R_t = \max\left\{\overline{R}\left(\frac{\Pi_{p,t}}{\overline{\Pi}_p}\right)^{\phi_\pi}, 1\right\}, \tag{A.10}$$

where parameter ϕ_π determines the response of the central bank to deviations of inflation from its steady state level.

Equilibrium. In this economy all production is consumed

$$Y_t = C_t. \tag{A.11}$$

The relation between aggregate output and employment can be written as[1]

$$N_t = \Delta_{w,t}\Delta_{p,t}Y_t^{\frac{1}{1-\alpha}}, \tag{A.12}$$

where $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon_p}{1-\alpha}} di$.

Finally, we assume bonds are in zero net supply. Hence, equilibrium in the bonds market requires

$$(1-\lambda)B_t^u + \lambda B_t^c = 0. \tag{A.13}$$

Since constrained agents have no access to financial markets, the last expression implies $B_t^u = B_t^c = 0$.

A.2 Price and Wage Setting *à* la Calvo

Firms face price stickiness *à* la Calvo. Hence, in every period, they reset prices with probability $1-\theta_p$. A firm that is able to reset prices in period t, chooses the price P_t^* that maximizes the following sum of discounted profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k \left\{ Q_{t,t+k}\left(P_t^* Y_{t+k|t} - TC_{t+k}(Y_{t+k|t})\right)\right\}, \tag{A.14}$$

[1] See Appendix B.1 for the derivation.

subject to the demand constraint given by

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p} Y_{t+k}, \tag{A.15}$$

which is the demand faced in $t+k$ by a firm that sets its price optimally in t. Total cost of producing $Y_{t+k|t}$ units is $TC_{t+k}(Y_{t+k|t}) \equiv W_{t+k} \left(\frac{Y_{t+k|t}}{A_{t+k}} \right)^{\frac{1}{1-\alpha}}$. $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}^u}{C_{t+k}^u} \right)^{-\sigma}$ is the stochastic discount factor, which depends only on the consumption of the unconstrained agents. The first-order condition for profits maximization reads

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M}^p MC_{t+k|t} \right) \right\} = 0, \tag{A.16}$$

where $\mathcal{M}^p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ is the desired markup and $MC_{t+k|t}$ is the nominal marginal cost.

The wage for each labor variety is set by a union operating in a monopolistically competitive market. Unions choose the wage rate that maximizes a weighted average of unconstrained and constrained lifetime utility, given by

$$\begin{aligned} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \chi_{t+k} \Bigg[(1-\lambda) \Bigg(\frac{(C_{t+k}^u - 1)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{(N_{t+k|t}^u)^{1+\varphi}}{1+\varphi} dj \Bigg) \\ + \lambda \Bigg(\frac{(C_{t+k}^c - 1)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{(N_{t+k|t}^c)^{1+\varphi}}{1+\varphi} dj \Bigg) \Bigg], \end{aligned} \tag{A.17}$$

subject to households' resource constraint and the sequence of demands for the labor variety they represent, given by

$$N_{t+k|t}^K = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\epsilon_w} N_{t+k}^K, \tag{A.18}$$

where W^* is the optimal wage chosen by a union that last resets its wage at t, $N_{t+k|t}^K$ is labor supply for the household's members whose wage was last reoptimized in period t and ϵ_w is the elasticity of substitution among labor varieties.

Assuming firms demand for constrained and unconstrained workers labor is the same, i.e.,

$N_t^u(j) = N_t^c(j) = N_t(j)$, the first-order condition of the union is

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \chi_{t+k} N_{t+k|t}^{1+\varphi} \left[\left((1-\lambda) \frac{1}{(C_{t+k}^u)^\sigma N_{t+k|t}^\varphi} + \lambda \frac{1}{(C_{t+k}^c)^\sigma N_{t+k|t}^\varphi} \right) \frac{W_t^*}{P_{t+k}} - \mathcal{M}^w \right] = 0, \quad \text{(A.19)}$$

where $\mathcal{M}^w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$ is the desired markup and $\Pi_{w,t} \equiv \frac{W_t}{W_{t-1}}$ is the gross inflation rate of wages.

B Proofs and derivations

B.1 Aggregation

Total labor supply must be equal to total demand. This is, $N_t = \int_0^1 \int_0^1 N_t(i,j) di dj$, where i denotes firms and j labor varieties. From the demand of each firm i for variety j we have

$$N_t = \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} \left(\int_0^1 N_t(i) di \right) dj.$$

Recalling the demand for each firm i's variety, $Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon_p} Y_t$, and from the production function of each firm i, $N_t(i) = \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}}$, we have

$$\begin{aligned} N_t &= \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} \left(\int_0^1 \left(\left(\frac{Y_t}{A_t} \right) \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_p} \right)^{\frac{1}{1-\alpha}} di \right) dj \\ &= \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \underbrace{\left(\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon_p}{1-\alpha}} di \right)}_{\equiv \Delta_{p,t}} \underbrace{\left(\int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} dj \right)}_{\equiv \Delta_{w,t}}, \end{aligned}$$

which is the same expression as in the main text.

B.2 Proof of Proposition 1

We separate the proof in three parts. First, we describe the evolution of price inflation. Second, we describe the labor supply in the economy with heterogeneity and wage rigidities. Finally, we describe the process for real wages.

Part 1: Price inflation

There is a mass $1-\theta_p$ of optimizing firms (denoted by superindex o) while the remainder θ_p set prices before the shock is realized (denoted by superindex m).

We start by describing the price setting problem optimizers face. Given the (log-linearized) demand function for a given variety i

$$y_t^o(i) = y_t - \epsilon_p(p_t^o(i) - p_t), \tag{B.1}$$

firms maximize profits setting their price as a markup μ^p over the marginal cost. Optimal pricing implies

$$p_t^o(i) = \mu^p + w_t - \log(1-\alpha) + \frac{\alpha}{1-\alpha} y_t^o(i), \tag{B.2}$$

where $\mu^p \equiv \log\left(\mathcal{M}^p\right)$. Substituting the demand (B.1) into the firm optimality condition (B.2) and rearranging yields

$$p_t^o(i) = \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\left(\mu^p + \omega_t - \log(1-\alpha) + \frac{\alpha}{1-\alpha} y_t\right) + p_t, \tag{B.3}$$

where $\omega_t \equiv w_t - p_t$ is the real wage.

Consider next the price setting problem that non-optimizers face. These firms set prices at the end of period $t-1$, and hence, they make their pricing decisions for period t based on the information set available at $t-1$. Optimization implies

$$p_t^m(i) = \mathbb{E}_{t-1}\left(\mu^p + w_t - \log(1-\alpha) + \alpha n_t^m(i)\right). \tag{B.4}$$

On the other hand, the aggregate price index is given by

$$P_t \equiv \left(\int_0^1 P_t(j)^{1-\epsilon_p} dj \right)^{\frac{1}{1-\epsilon_p}},$$

which implies

$$P_t = \left((1-\theta_p)(P_t^o)^{1-\epsilon_p} + \theta_p (P_t^m)^{1-\epsilon_p} \right)^{\frac{1}{1-\epsilon_p}}.$$

Accordingly, the following relation holds around steady state

$$p_t = (1-\theta_p) p_t^o + \theta_p p_t^m.$$

Substituting the pricing rules from optimizers (B.3) and non-optimizers (B.4) into the aggregate price index yields the following price inflation equation

$$\widehat{\pi}_t^p = \kappa_\pi \left(\widehat{\omega}_t + \frac{\alpha}{1-\alpha} \widehat{y}_t \right) + \mathbb{E}_{t-1} \widehat{x}_t^p, \tag{B.5}$$

where hat variables correspond to log-deviations with respect to steady-state, $\widehat{x}_t^p \equiv \widehat{\omega}_t + \alpha \widehat{n}_t^m(j) + \widehat{\pi}_t^p$ and $\kappa_\pi \equiv \frac{1-\theta_p}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$.

Part 2: Labor supply

Before solving for the wage schedule, we first solve for the labor supply. This is needed since the labor supply in our model with inequality depends on the consumption gap as it depends on the average marginal utility (thus, depends on both constrained and unconstrained consumption). In our setting, unions maximize households utility by setting the wage (in real terms) to be a markup μ^w over the marginal rate of substitution. However, in a heterogeneous economy like ours, the average marginal utility depends on the fraction of each group of consumers, in particular, it depends on the share of constrained consumers λ. That is why it does not directly depend on the output gap as usual.

To start, we compute three intermediate results. First we show how the average marginal utility of consumption depends on inequality (λ) and the level of consumption of each group. Then we show the relation between individual constrained and unconstrained consumption with output and the consumption gap. Finally, we show how average marginal utility of consumption depends on output and real wages.

Lemma 1. *The average marginal utility of consumption, U, as a function of individual consumptions can be approximated as*

$$\widehat{u}_t = -\sigma(u_c \widehat{c}_t^c + u_u \widehat{c}_t^u), \tag{B.6}$$

with $u_c \equiv \frac{\lambda}{\lambda+(1-\lambda)\gamma^{-\sigma}}$ *and* $u_u \equiv 1 - u_c$, *and* $\gamma \equiv \frac{C^U}{C^C}$ *being the steady-state consumption gap.*

Proof. The marginal utility of consumption of agent $K \in \{c, u\}$ is $(C_t^K)^{-\sigma}$. therefore, the average marginal utility of consumption, U, can be written as $U = \lambda(C_t^c)^{-\sigma} + (1-\lambda)(C_t^u)^{-\sigma}$. Taking a first order approximation around steady-state, we get $\widehat{u}_t = -\sigma(u_c \widehat{c}_t^c + u_u \widehat{c}_t^u)$, with $u_c \equiv \frac{\lambda C_c^{-\sigma}}{\lambda C_c^{-\sigma}+(1-\lambda)C_u^{-\sigma}}$. Replacing $C_u = \gamma C_c$ we get $u_c \equiv \frac{\lambda}{\lambda+(1-\lambda)\gamma^{-\sigma}}$ and $u_u \equiv 1 - u_c$. □

Lemma 1 shows that the average marginal utility of consumption, which is a relevant piece of information for unions to set nominal wages and total hours, depends on the fraction of constrained agents, the income effect on labor supply (given by σ) and the steady-state consumption gap, γ. Clearly, whenever $\lambda = 0$ ($\lambda = 1$), $\widehat{c}_t^c = \widehat{c}_t^u = \widehat{c}_t$, the consumption gap is zero and $u_u = 1$ ($u_u = 0$), so $\widehat{u}_t = -\sigma\widehat{c}_t$, which is the same marginal utility of consumption of a representative agent model.

Lemma 2 shows the aggregate relation between consumption on each segment of the population, output and the consumption gap.

Lemma 2. *Consumption of constrained and unconstrained agents can be approximated as*

$$\widehat{c}_t^c = \widehat{y}_t - \frac{(1-\lambda)\gamma}{\lambda+(1-\lambda)\gamma}\widehat{\gamma}_t,$$

$$\widehat{c}_t^u = \widehat{y}_t + \frac{\lambda}{\lambda+(1-\lambda)\gamma}\widehat{\gamma}_t.$$

Proof. From market clearing in the market of final goods and the definition of aggregate consumption we have $Y_t = C_t = \lambda C_t^c + (1-\lambda)C_t^u$. Using the definition of the consumption gap, the previous expression is $Y_t = C_t^U\Big(\frac{\lambda}{\gamma_t} + 1 - \lambda\Big)$. This can be approximated as $\widehat{y}_t = \widehat{c}_t^u - \frac{\lambda}{(1-\lambda)\gamma+\lambda}\widehat{\gamma}_t$. On the other hand, the consumption gap is $\widehat{\gamma}_t = \widehat{c}_t^u - \widehat{c}_t^c$. Using these two equations to solve for constrained and unconstrained consumption as a function of output and the consumption gap gives the desired equations. □

Finally, Lemma 3 fully characterizes the behavior of the average marginal utility of consumption as a function of output and real wages.

Lemma 3. *The average marginal utility of consumption can be written in terms of output and real wage as*

$$\widehat{u}_t = -\varpi_1\widehat{y}_t - \varpi_2\widehat{\omega}_t, \tag{B.7}$$

with $\varpi_1 \equiv \sigma + \overline{u}\Psi\frac{\alpha}{1-\alpha}$ *and* $\varpi_2 \equiv \overline{u}\Psi$*, and where* $\overline{u} \equiv -\sigma\frac{\lambda(1-\lambda)\gamma^{-\sigma}-\lambda(1-\lambda)\gamma}{[(1-\lambda)\gamma+\lambda][\lambda+(1-\lambda)\gamma^{-\sigma}]}$ *and* $\Psi \equiv \frac{\mathcal{M}^p}{(1-\lambda)\big(1-\alpha+\frac{1}{1-\lambda}(\mathcal{M}^p-(1-\alpha))\big)}$ *comes from the relation between consumption inequality and price markup in Equation (6).*

Proof. First note that by combining the results from Lemma 1 and Lemma 2, we get $\widehat{u}_t = -\sigma\widehat{y}_t + \overline{u}\widehat{\gamma}_t$, with $\overline{u}$ previously defined. Then, replacing Equation (7) into (6) (average price markup into consumption gap), we get $\widehat{\gamma}_t = \Psi\Big(-\frac{\alpha}{1-\alpha}\widehat{y}_t - \widehat{\omega}_t\Big)$. Combining these results and re-arranging, we get equation (B.7). □

Now we are ready to solve for the actual labor supply schedule. The optimality condition for the union is $\frac{W_t}{P_t}\Big(\frac{\lambda}{(C_t^c)^{-\sigma}} + \frac{1-\lambda}{(C_t^u)^{-\sigma}}\Big) = \mathcal{M}^w N_t^{\varphi}$, where $\mathcal{M}^w$ is the wage markup and we impose the condition that all workers have the same wage and the same number of hours. Note that the term in parentheses is the average marginal utility of consumption across workers. Taking a log-linear approximation, we get $\widehat{\omega}_t + \widehat{u}_t = \varphi\widehat{n}_t$. Replacing the average marginal utility of consumption (B.7), we obtain

$$\widehat{\omega}_t - \varpi_1 \widehat{y}_t - \varpi_2 \widehat{\omega}_t = \varphi \widehat{n}_t.$$

Re-ordering and defining $\varpi \equiv \frac{\varpi_1}{1-\varpi_2}$ and $\overline{\varphi} \equiv \frac{\varphi}{1-\varpi_2}$, we get the average labor supply

$$\widehat{\omega}_t = \overline{\varphi} \widehat{n}_t + \varpi \widehat{y}_t, \tag{B.8}$$

where $\overline{\varphi}$ and ϖ are parameters that depend on inequality, λ.

Part 3: Real wage

Finally, we can describe the evolution of real wages. As in the case of price inflation, we can separate the problem between optimizers and non-optimizers. Consider first the optimizers problem. Unions optimaly set the wage according to

$$\widehat{\omega}_t^o(j) - \varpi \widehat{y}_t = \overline{\varphi} \widehat{n}_t^o(j). \tag{B.9}$$

Given the demand function

$$n_t^o(j) = n_t - \epsilon_w(\omega_t^o(j) - \omega_t), \tag{B.10}$$

we obtain

$$\widehat{\omega}_t^o(j) = \frac{1}{1+\varphi\epsilon_w}(\varpi \widehat{y}_t + \overline{\varphi} \widehat{n}_t + \varphi \epsilon_w \widehat{\omega}_t). \tag{B.11}$$

Consider next the wage setting problem non-optimizers face. Identical to the firms problem, non-optimizing unions decide wages for period t based on the information set available at $t-1$, implying

$$\mathbb{E}_{t-1}\left(\hat{\omega}_t^m(j) - \varpi \hat{y}_t - \overline{\varphi} \hat{n}_t^m(j)\right) = 0,$$

which can be rewritten as

$$\widehat{\omega}_t^m(j) = -\widehat{\pi}_t^p + \mathbb{E}_{t-1}\left(\varpi \hat{y}_t + \overline{\varphi} \widehat{n}_t^m(j) + \widehat{\pi}_t^p\right). \tag{B.12}$$

On the other hand, the aggregate wage is given by

$$W_t \equiv \left(\int_0^1 W_t(i)^{1-\epsilon_w} di \right)^{\frac{1}{1-\epsilon_w}},$$

implying

$$W_t = \left(\theta_w (W_t^m)^{1-\epsilon_w} + (1-\theta_w)(W_t^o)^{1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}}.$$

The previous expression can be written in log-deviation from the steady state as

$$\widehat{\omega}_t = \theta_w \widehat{\omega}_t^m + (1-\theta_w)\widehat{\omega}_t^o.$$

Finally, substituting optimizers and non-optimizers wage setting rules (B.11) and (B.12) into the aggregate wage equation and imposing market clearing yields

$$\widehat{\omega}_t = \kappa_\omega(\varpi \widehat{y}_t + \overline{\varphi} \widehat{n}_t) - \varsigma \widehat{\pi}_t^p + \mathbb{E}_{t-1}\widehat{x}_t^w, \tag{B.13}$$

where $\kappa_\omega \equiv \frac{1-\theta_w}{1+\theta_w \varphi \epsilon_w}$, $\varsigma \equiv \frac{\theta_w(1+\varphi\epsilon_w)}{1+\theta_w\varphi\epsilon_w}$ and $\widehat{x}_t^w \equiv \varsigma\left(\varpi \widehat{y}_t + \overline{\varphi}\widehat{n}_t + \widehat{\pi}_t^p\right)$.

B.3 Proof of Proposition 3

We can show that $\lim_{\theta_p \to 0} \Xi = -\frac{\alpha}{1-\alpha}$ and $\frac{\partial \Xi}{\partial \theta_p} > 0$.[2] Accordingly, for any parameter calibration we have that $\Xi \geqslant -\frac{\alpha}{1-\alpha}$. Additionally, we have that $\Xi = \varpi + \frac{\overline{\varphi}}{1-\alpha}$ when $\theta_w \to 0$, and $\frac{\partial \Xi_y}{\partial \theta_w} < 0$.[3] It follows that for any parametrization $\Xi \leqslant \varpi + \frac{\overline{\varphi}}{1-\alpha}$. Therefore, $\varpi + \frac{\overline{\varphi}}{1-\alpha} \geqslant \Xi_y \geqslant -\frac{\alpha}{1-\alpha}$.

B.4 Derivation of the IS equation

From (A.5) we can get

$$C_t = C_t^u \left((1-\lambda) + \lambda \frac{1}{\gamma_t} \right),$$

which can be rewritten in log-deviation from steady state as

$$\widehat{c}_t = \widehat{c}_t^u - \frac{\lambda}{(1-\lambda)\gamma + \lambda} \widehat{\gamma}_t. \tag{B.14}$$

On the other hand, the Euler equation of unconstrained agents is $\widehat{c}_t = \mathbb{E}_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma}\mathbb{E}_t[\widehat{r}_t - \widehat{\pi}^p_{t+1} - (1-\rho_\chi)\chi_t]$. Replacing (12) and (B.14) into the previous expression and imposing market clearing we obtain

$$\begin{aligned}
&\widehat{y}_t + \frac{\lambda}{(1-\lambda)\gamma + \lambda}(-\Theta \widehat{y}_t - \Psi \mathbb{E}_{t-1}\widehat{x}_t) \\
&\quad = \mathbb{E}_t \left[\widehat{y}_{t+1} + \frac{\lambda}{(1-\lambda)\gamma + \lambda}(-\Theta \widehat{y}_{t+1} - \Psi \mathbb{E}_t \widehat{x}_{t+1}) \right] - \frac{1}{\sigma}\mathbb{E}_t(\widehat{r}_t - \widehat{\pi}^p_{t+1} - (1-\rho_\chi)\widehat{\chi}_t).
\end{aligned}$$

Assume next that the economy starts at steady state in period $t-1$ and that shocks are iid. Since shocks are unexpected, then at $t-1$ agents forecast that the economy at t will remain at steady state, i.e., $\mathbb{E}_{t-1}\widehat{x}_t = 0$. Additionally, since shocks have no persistence, all real variables return

[2] The derivative is given by $\frac{\partial \Xi}{\partial \theta_p} = \frac{\left(\frac{\alpha}{1-\alpha} + \frac{1-\theta_w}{1+\theta_w \overline{\varphi}\epsilon_w}\left(\varpi + \frac{\overline{\varphi}}{1-\alpha}\right)\right)\frac{\theta_w(1+\overline{\varphi}\epsilon_w)}{1+\theta_w\overline{\varphi}\epsilon_w}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\frac{1}{\theta_p^2}}{\left(1+\frac{\theta_w(1+\overline{\varphi}\epsilon_w)}{1+\theta_w\overline{\varphi}\epsilon_w}\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\right)^2} > 0.$

[3] In particular, $\frac{\partial \Xi}{\partial \theta_w} = -\frac{\left(\left(\varpi + \frac{\overline{\varphi}}{1-\alpha}\right) + (1+\overline{\varphi}\epsilon_w)\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\frac{\alpha}{1-\alpha}\right) + \left(\varpi + \frac{\overline{\varphi}}{1-\alpha}\right)\left(\overline{\varphi}\epsilon_w + (1+\overline{\varphi}\epsilon_w)\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\right)}{\left((1+\theta_w\overline{\varphi}\epsilon_w) + \theta_w(1+\overline{\varphi}\epsilon_w)\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\right)^2} < 0.$

to steady state at $t+1$.[4] The nominal variables at $t+1$ on the other hand will be determined by monetary policy. We assume the central bank implements a policy such that $\widehat{\pi}^p_{t+1}=0$. Accordingly, we have $\mathbb{E}_t\widehat{y}_{t+1}=\mathbb{E}_t\widehat{\pi}^p_{t+1}=\mathbb{E}_t\widehat{x}_{t+1}=0$, and hence the aggregate Euler equation can be written as

$$\widehat{y}_t=-\frac{1}{\sigma}\frac{1}{1-\frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta}\mathbb{E}_t\left(\widehat{r}_t-\widehat{\chi}_t\right). \tag{B.15}$$

Expression (B.15) is the Euler equation under our simplifying assumptions. Notice that, contrary to the RANK case in which $\lambda=0$ and the slope of the IS is $-1/\sigma$, the response of output to the interest rate also depends on the market incompleteness parameter λ, through parameter Θ, which governs the cyclicality of the consumption gap.

B.5 Proof of Proposition 7

Given $\Xi\geqslant-\frac{\alpha}{1-\alpha}$ and $\Psi>0$, it follows that $\Theta\equiv\Psi\left(\Xi+\frac{\alpha}{1-\alpha}\right)\geqslant 0$. This above implies $\Lambda\equiv\frac{1}{1-\frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta}\notin(0,1)$.

B.6 Proof of Proposition 8

From (13), the slope of the IS curve is given by $-\frac{1}{\sigma}\Lambda$. Inversion of the IS thus requires $\Lambda\equiv\frac{1}{1-\frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta}<0$, or, equivalently, $\frac{\lambda}{(1-\lambda)\gamma+\lambda}\Theta>1$. Substituting Θ, it can be shown that the above relation holds whenever $\lambda>\frac{\frac{\mathcal{M}^p}{1-\alpha}}{\frac{\kappa\omega\left(\sigma+\frac{\varphi}{1-\alpha}\right)}{1+\varsigma\kappa_\pi}+\frac{1}{1+\varsigma\kappa_\pi}\frac{\alpha}{1-\alpha}+1}$.[5]

C Welfare losses

We derive a general welfare loss function for the economy, taking into account limited asset market participation. We assume that the central bank seeks to minimize the weighted utility of constrained and unconstrained agents (with weights given by their relative sizes).[6] Taking a second order

[4] At t agents correctly anticipate no shocks at $t+1$, hence it is as if the economy was not affected by any friction at $t+1$ and thus the allocation must coincide with that of the flexible price economy. The latter ensures all real variables return to steady state at $t+1$.

[5] To facilitate comparability of our results with existing literature, this derivation assume labor unions set wages to maximize the utility of the average household.

[6] For simplicity, we further assume the existence of a labor subsidy that corrects for the inefficiencies generated by monopolistic competition, and transfers that equate the steady state consumption of constrained and unconstrained households.

approximation of utility around the efficient steady state with no inequality, average welfare losses can be expressed as

$$L = \frac{1}{2}\left[\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) var(\tilde{y}_t) + \sigma\lambda(1-\lambda) var(\widehat{\gamma}_t) + \frac{\epsilon_p}{\lambda_p} var(\pi_t^p) + \frac{(1-\alpha)\epsilon_w}{\lambda_w} var(\pi_t^w)\right], \tag{C.1}$$

where $\lambda_p \equiv \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$ and $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\varphi\epsilon_w)}$. Welfare losses are a function of the output gap, price and wage inflation volatility, and the consumption gap. The latter term captures inequality and arises from the existence of limited asset markets participation. This is the metric we use to analyze the gains from wage flexibility.

D The slope of the IS conditional on monetary policy shocks

A way to write the slope of the IS (denoted by S) is

$$S = -\frac{(1-\alpha)(1+\varsigma\kappa_\pi)}{\sigma(1-\alpha)(1+\varsigma\kappa_\pi) + (\alpha+\kappa_\omega x)\Omega^*}, \tag{D.1}$$

with $\Omega^* = -\sigma\Gamma\Psi = -\sigma\frac{\lambda(\epsilon_p-1)(1-\alpha)}{\epsilon_p-\lambda(\epsilon_p-1)(1-\alpha)}$ and $x = \varpi(1-\alpha)+\overline{\varphi}$. Then, we compute the derivative of this slope, $\frac{\partial S}{\partial\theta_p}$, as

$$\frac{\partial S}{\partial\theta_p} = -(1-\alpha)\varsigma(\alpha+\kappa_w x)\sigma\frac{\lambda(\epsilon_p-1)(1-\alpha)}{\epsilon_p-\lambda(\epsilon_p-1)(1-\alpha)}\frac{(1-\alpha)}{1-\alpha+\alpha\epsilon_w}. \tag{D.2}$$

Notice that as $x > 0$, and all the remaining terms are positive, the slope of the IS increases with the price rigidity in absolute value. This is, output through the IS equation is more volatile conditional on monetary policy shocks.

E Computing the threshold $\overline{\phi}_\omega$

We have

$$S = -\frac{(1-\alpha)(1+\varsigma\kappa_\pi)}{\sigma(1-\alpha)(1+\varsigma\kappa_\pi) + (\alpha+\kappa_\omega x)\tilde{\Omega} + \phi_\omega(\kappa_\omega x - \varsigma\kappa_\pi\alpha)}, \tag{E.1}$$

with $\tilde{\Omega} = \kappa_\pi(\phi_\omega + \phi_\pi) - \sigma\Gamma\Psi$ and $\Gamma\Psi = \frac{\lambda(\epsilon_p-1)(1-\alpha)}{\epsilon_p-\lambda(\epsilon_p-1)(1-\alpha)}$. The partial derivative with respect to θ_w is

$$\frac{\partial S}{\partial \theta_\omega} = -\left\{\kappa_\pi(\alpha + \kappa_\omega x) + (1 + \varsigma\kappa_\pi)x\right\}\left(\tilde{\Omega} + \phi_\omega\right)\frac{1-\alpha}{D^2}\frac{\partial \kappa_\omega}{\partial \theta_\omega}, \tag{E.2}$$

where D denotes the denominator of the slope of the IS curve. Notice that $\varsigma = 1 - \kappa_\omega$, then

$$\frac{\partial S}{\partial \theta_\omega} = -\left\{\kappa_\pi(\alpha + x) + x\right\}\left(\tilde{\Omega} + \phi_\omega\right)\frac{1-\alpha}{D^2}\frac{\partial \kappa_\omega}{\partial \theta_\omega}. \tag{E.3}$$

with $\{\kappa_\pi(\alpha + x) + x\} > 0$, $\frac{1-\alpha}{D^2} > 0$, and $\frac{\partial \kappa_\omega}{\partial \theta_\omega} < 0$ for any κ_π and λ. Then, we need to analyze $\left(\tilde{\Omega} + \phi_\omega\right)$, which sign depends on the degree of price stickiness through κ_π. Recall that $\Gamma\Psi = \frac{\lambda(\epsilon_p-1)(1-\alpha)}{\epsilon_p-\lambda(\epsilon_p-1)(1-\alpha)}$, and $\kappa_\pi = \frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$, then

$$\tilde{\Omega} + \phi_\omega = \frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}(\phi_\pi + \phi_\omega) - \sigma\frac{\lambda(\epsilon_p - 1)(1-\alpha)}{\epsilon_p - \lambda(\epsilon_p - 1)(1-\alpha)} + \phi_\omega. \tag{E.4}$$

Hence, the threshold is given by the following expression

$$\frac{1-\theta_p}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\phi_\pi + \frac{1}{\theta_p}\frac{1-\alpha+\theta_p\alpha\epsilon_p}{1-\alpha+\alpha\epsilon_p}\phi_\omega = \sigma\frac{\lambda(\epsilon_p - 1)(1-\alpha)}{\epsilon_p - \lambda(\epsilon_p - 1)(1-\alpha)}, \tag{E.5}$$

$$\overline{\phi}_\omega = \frac{\sigma\lambda(\epsilon_p - 1)(1-\alpha)(1-\alpha+\alpha\epsilon_p)\theta_p}{(\epsilon_p - \lambda(\epsilon_p - 1)(1-\alpha))(1-\alpha+\theta_p\alpha\epsilon_p)} - \frac{(1-\theta_p)(1-\alpha)}{1-\alpha+\theta_p\alpha\epsilon_p}\phi_\pi. \tag{E.6}$$

Now we can obtain $\overline{\phi}_\pi$, which is given by

$$\overline{\phi}_\pi = \frac{\sigma\lambda(\epsilon_p - 1)(1-\alpha+\alpha\epsilon_p)}{\epsilon_p - \lambda(\epsilon_p - 1)(1-\alpha)}\frac{\theta_p}{(1-\theta_p)} - \frac{1-\alpha+\alpha\epsilon_p\theta_p}{(1-\theta_p)(1-\alpha)}\phi_\omega. \tag{E.7}$$

F Comparison with Ascari et al. (2017)

The main difference between this paper and that of Ascari et al. (2017) is that we include–and analyze– explicitly the fact that the real wage depends on price inflation, while Ascari et al. (2017) does not. That paper takes into account that the real wage depends on different timings between prices and wage adjustments, but they do not explore explicitly the fact that the real wage depends on price inflation, and hence on the output gap. To show that their setup also has the features we emphasize in our paper, take their real wages under sticky prices equation (equation A33 in their appendix)

$$\omega_t = \frac{1}{1+\beta+\kappa_w}[w_{t-1} - p_t] + \frac{\beta}{1+\beta+\kappa_w}\mathbb{E}_t[w_{t+1} - p_t] + \frac{\kappa_w}{1+\beta+\kappa_w}((\sigma+\varphi)x_t + a_t) \tag{F.1}$$

where ω_t is real wages, β the discount factor, p_t aggregate prices, κ_w the slope of the wage Phillips curve, σ the inverse of the IES, φ the inverse of the Frisch elasticity, x_t the output gap and a_t a technology shock.

Taking the first difference of Equation (F.1) and replacing into the IS equation (assuming no technology shocks, $a_t = 0 \ \forall \ t$), we get

$$\Delta\omega_{t+1} = F + \frac{\kappa_w}{1+\beta+\kappa_w}((\sigma+\varphi)\Delta x_t) \tag{F.2}$$
$$F = \frac{1}{1+\beta+\kappa_w}[\pi_t^w + \pi_{t+1}] + \frac{\beta}{1+\beta+\kappa_w}\mathbb{E}_t[\pi_{t+2}^w - \pi_{t+1}]$$

Another way to express real wages in their setup–to make them depend *explicitly* on price inflation–is

$$\omega_t = \frac{1}{1+\beta+\kappa_w}[\omega_{t-1} - \pi_t] + \frac{\beta}{1+\beta+\kappa_w}\mathbb{E}_t[\omega_{t+1} + \pi_{t+1}] + \frac{\kappa_w}{1+\beta+\kappa_w}((\sigma+\varphi)x_t) \tag{F.3}$$

where now, the real wage goes down if inflation is higher, which is consistent with the channel we emphasize in our paper. Moreover, taking into account that $\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa_p\omega_t$ in Ascari et al. (2017), the previous equation boils down to

$$\omega_t = \frac{\kappa_w}{1+\beta+\kappa_w+\kappa_p}((\sigma+\varphi)x_t) + \Theta_t, \tag{F.4}$$

with $\Theta_t = \frac{\omega_{t-1}+\beta\mathbb{E}_t[\omega_{t+1}]}{1+\beta+\kappa_w+\kappa_p}$.

Take the IS curve

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\mathbb{E}_t(i_t - \pi_{t+1} - r_t^{Eff}) - \frac{\lambda}{1-\lambda}\mathbb{E}_t \Delta\omega_{t+1}, \tag{F.5}$$

gives

$$x_t = \mathbb{E}_t x_{t+1} - \delta\mathbb{E}_t(i_t - \pi_{t+1} - r_t^{Eff}) - \delta\Delta\Theta_{t+1} \tag{F.6}$$

$$\delta = \frac{1}{\sigma}\left[1 - \frac{\lambda}{1-\lambda}\frac{\kappa_w(\sigma+\varphi)}{1+\beta+\kappa_w+\kappa_p}\right]^{-1}.$$

This equation clearly shows that the effects of wage flexibility crucially depend on the degree of price flexibility. As we argue in the body of the paper, there is a lower impact of wage rigidities when prices are more flexible. The slope of Ascari et al. (2017) is the right only if prices are fully sticky, i.e. $\kappa_p = 0$.

References

Ascari, G., A. Colciago, and L. Rossi (2017): "Limited Asset Market Participation, Sticky Wages, and Monetary Policy," *Economic Inquiry*, 55, 878–897.