Production Linkages and Nominal Rigidities in a Small Open Economy[*]

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Abstract

This paper investigates how production linkages and nominal rigidities shape the impact of shocks in a commodity-exporting small open economy. We develop a multi-sector New Keynesian model in which domestic firms utilize labor, materials, and imported inputs, whose prices may be sticky, leading to incomplete exchange-rate pass-through. Analytically, we demonstrate that a rise in commodity prices can be inflationary or disinflationary depending on whether rigidity is focused on domestic or import prices. Quantitatively, using Chilean data, we find that network linkages reduce responsiveness and alter the threshold where the sign of inflation changes, having broader implications for policy responses to external shocks.

JEL Codes: E30, E32, F41

Keywords: Emerging economies, Business cycles, Multi-sector Calvo model, Input-output linkages

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1 Introduction

Commodity-exporting small open economies (SOEs) are vulnerable to international shocks. Changes in commodity prices, global interest rates, or foreign demand influence domestic activity through trade and income channels, as well as through shifts in relative prices, such as the exchange rate. Simultaneously, domestic production is structured via dense input-output (IO) linkages that spread shocks across sectors, while nominal rigidities slow down the adjustment of relative prices. These characteristics mean that the macroeconomic effects of external shocks depend not just on their source but also on where nominal rigidities are located within the production network. Understanding these processes is essential for analyzing past commodity-related fluctuations and for shaping monetary policy in resource-rich SOEs.

Recent literature has emphasized the importance of production networks in shaping aggregate fluctuations. Gabaix (2011) and Acemoglu et al. (2012) showed how micro shocks can aggregate into sizable macro volatility through highly interconnected sectors. More recently, Pasten et al. (2020) demonstrated that nominal rigidities interact with production linkages to amplify and distort the transmission of shocks in a closed economy. In their model, the distribution of sectoral stickiness determines the strength of network amplification. However, their analysis mainly focuses on closed economies. In small open economies, imported inputs are key nodes in the production process, and their prices are influenced by exchange-rate movements and the degree of pass-through from foreign producers. This distinction is crucial as it changes both the direction and strength of inflation and output responses to external shocks.

Our paper builds on this insight. We develop a multi-sector New Keynesian model for a commodity-exporting SOE in which firms produce using labor, domestic inputs, and an imported input. Crucially, imported input prices are determined by monopolistic firms facing nominal rigidities. This results in incomplete and delayed exchange-rate pass-through at business-cycle frequencies, aligning with empirical evidence (e.g., Campa and Goldberg, 2005; Burstein and Gopinath, 2014). This modeling choice distinguishes our analysis from the closed-economy network literature and brings the theory closer to the data: exchange-rate pass-through into import prices is a well-documented but under-explored source of heterogeneity across SOEs.[1]

We show analytically that a rise in commodity prices triggers two opposing cost channels. On one hand, higher commodity income and real exchange-rate appreciation increase wages and

[1] Smets and Wouters (2002), Monacelli (2005), and Garcia-Schmidt and Garcia-Cicco (2020) incorporate these nominal rigidities but do not study their aggregate effects.

domestic costs (a wage-push channel). On the other hand, cheaper imported inputs reduce costs through a real exchange rate appreciation (an import-cost relief channel). Which channel prevails depends on where nominal rigidity is concentrated. If import prices are sticky, the exchange-rate appreciation does not pass through quickly, the wage-push channel dominates, and inflation increases. If import prices are flexible, imported input costs decrease, and inflation drops. Therefore, the economy behaves as if it is subject to either a positive demand or a positive supply shock. Input-output linkages amplify the dominant force, shifting the threshold at which inflation's direction reverses.

Our contribution is twofold. First, on the theoretical side, we derive closed-form results that clarify how relative sectoral rigidities influence the inflation-output trade-off. We present propositions showing that (i) flexible import prices reduce inflation on impact, while flexible domestic prices increase it; (ii) production linkages amplify whichever channel is dominant; and (iii) these outcomes are inherently open-economy phenomena, not present in closed-economy networks. Our analysis thus complements and extends Pasten et al. (2020) by demonstrating how imported inputs create a qualitatively new mechanism.

Second, on the quantitative side, we discipline the model using Chilean data. We calibrate sectoral input shares with Chile's IO table, calibrate nominal rigidities in domestic sectors to micro price data, and set import-price stickiness based on exchange-rate pass-through evidence. This calibration ensures that key parameters, especially import-price rigidity, are empirically grounded. We then examine responses to a commodity-price boom. Three main lessons emerge. (i) The sign of inflation on impact is not predetermined but depends on relative rigidities: sticky imports cause inflation, while flexible imports lead to disinflation. (ii) Stronger production linkages reduce output sensitivity but make inflation more responsive to the distribution of rigidities. (iii) The same logic applies to other shocks: those that directly affect marginal costs (e.g., TFP, import-price shocks) are most impacted, whereas demand shocks (e.g., foreign demand, global interest rates) are mainly amplified without changing sign.[2]

Our findings have policy implications. Conventional wisdom in SOEs often views commodity booms as inflationary demand shocks. Our results demonstrate that this is not a fundamental feature of terms-of-trade shocks but rather a result of where nominal rigidities are located. Although

[2]Our study of alternative shocks is motivated by the findings of Neumeyer and Perri (2005), Uribe and Yue (2006), Aguiar and Gopinath (2007), Dib (2008), Chang and Fernandez (2013), and Garcia-Schmidt and Garcia-Cicco (2020), among others.

these booms always increase output, the inflation response depends on relative nominal rigidities. In economies with sticky import prices, booms lead to inflation, but in economies with flexible import prices, the same booms result in disinflation. These differences underscore the importance of accurately measuring exchange-rate pass-through and developing policy frameworks that take into account how sectoral rigidities affect overall responses.

Related literature. This paper contributes to three branches of the literature. First, it connects to the extensive research on the role of terms-of-trade and commodity-price shocks in emerging economies. Early studies examined these shocks within real business cycle models, focusing on the wealth and substitution effects caused by relative price changes (Mendoza, 1995; Kose, 2002; Caputo and Irarrazaval, 2017; Drechsel and Tenreyro, 2018; Schmitt-Grohe and Uribe, 2018; Kohn et al., 2018). More recent research incorporates commodity shocks into economies with nominal rigidities, examining their implications for the conduct of monetary and fiscal policy. For instance, Catão and Chang (2015) analyzes optimal monetary policy in a commodity-importing economy, while Drechsel et al. (2019) emphasizes the collateral role of commodity output in easing financial frictions. Fornero et al. (2016) and Medina and Soto (2016) investigate fiscal responses and investment patterns in SOEs with sticky prices. Compared to this body of work, we shift the focus from policy formulation to the actual transmission mechanisms, emphasizing how the interaction between nominal rigidities and production linkages determines whether a commodity boom is inflationary or disinflationary.

Second, our work is closely related to papers emphasizing production linkages in the transmission of shocks. We build on Romero (2025), who studies how IO linkages shape the response of a SOE to a commodity price shock in a real business cycle model for a small open economy, to analyze how nominal rigidities shape the aggregate responses to the shock. Cao and Dong (2020) examines the role of domestic and international input-output linkages in the Canadian slowdown of 2014-2016, finding that production networks influence the impact of commodity shocks. Compared to them, we explore the systematic effects of such shocks in a small open economy New Keynesian model, and importantly, investigate the interaction with nominal rigidities. We demonstrate that the placement of price stickiness (import versus domestic sectors) is key in determining aggregate outcomes, a factor not considered in their analysis.

Third, we connect to the growing multi-sector New Keynesian literature that studies heterogeneous price stickiness and production networks. Pasten et al. (2020) highlights how variation in sectoral rigidity amplifies shocks in closed economies. Carvalho and Schwartzman (2015) and

Carvalho et al. (2021) investigate how heterogeneous nominal rigidities affect inflation dynamics. In contrast, Wei and Xie (2020) analyzes optimal monetary policy in models with networks for global value chains based on the canonical small open economy New Keynesian benchmark of Gali and Monacelli (2005). Our contribution extends these insights to an open-economy context where imported inputs face nominal frictions, leading to incomplete pass-through and a new source of amplification.

In this sense, our paper complements closed-economy network models by highlighting a channel that is missing from them: the tension between wage-push forces stemming from higher commodity income and import-cost relief forces resulting from exchange-rate appreciation. We demonstrate that the interaction between production linkages and the placement of nominal rigidities determines which force dominates, affecting both the direction and magnitude of inflation responses to external shocks. Two additional insights arise: (i) the importance of nominal rigidities and production linkages varies depending on the type of shock–cost-push shocks have the strongest interaction, while demand shocks operate mainly through amplification–and (ii) sectoral rigidities can either increase or reduce overall volatility, depending on whether they are in imported or domestic inputs.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 derives the analytical results, clarifying the role of relative rigidities and IO linkages. Section 4 quantifies the model for Chile and studies responses to different shocks. Section 5 concludes.

2 Model

The economy is characterized by a representative household, N domestic productive sectors indexed by j, an aggregator producing importable goods, and an aggregator producing exportable goods. The household consumes an aggregate good composed of the different domestic producers, supplies labor, and can take both domestic and foreign debt. The productive sectors are characterized by a competitive aggregator that combines a continuum of differentiated varieties subject to nominal rigidities. The output of each industry can be used for consumption, materials for other sectors, and exports. The importable sector has similar characteristics (monopolistic competition and nominal rigidities), but produces by demanding goods from abroad and transforming them into different varieties. The commodity sector, denoted by $j = 1$, is characterized by a representative competitive firm. Because the economy is small and open, the commodity sector takes the price of the commodity good as given.

2.1 Household

The representative household maximizes its lifetime utility by choosing consumption, domestic and foreign bond positions, and the amount of labor supplied to firms, subject to its budget constraint. Formally, it solves the following intertemporal problem

$$\max_{C_t, L_t, B_t, B_t^*} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \kappa_t \frac{L_t^{1+\varphi}}{1+\varphi} \right)$$

subject to

$$P_t C_t + B_t + \mathcal{E}_t B_t^* = W_t L_t + R_{t-1} B_{t-1} + \mathcal{E}_t R_{t-1}^* B_{t-1} + D_t, \tag{1}$$

where $\sigma \geq 1$ is the intertemporal elasticity of substitution, β is the discount factor, $\varphi \geq 0$ is the inverse of the Frisch elasticity, and $\kappa_t = \overline{\kappa} C_t^{-\sigma}$ governs the disutility of labor and eliminate the wealth effect on the supply of labor while keeping separability between consumption and labor (Gali et al., 2012). In terms of variables, C_t is the aggregate consumption bundle (with price P_t) and L_t denotes total hours offered by the household at a wage rate W_t. In this open economy context, $\mathcal{E}_t$ denotes the nominal exchange rate, while B_t and B_t^* are the amount of domestic and foreign bonds demanded at period t, respectively, with gross nominal return R_t and R_t^*. The term D_t collects all profits in the economy.

Intratemporal decisions. Each period, the household must allocate its consumption expenditures across different domestic sectors. The consumption aggregator C_t takes a Cobb-Douglas form

$$C_t = \delta_c \prod_{j=1}^{N} C_{jt}^{\vartheta_j}, \quad \text{with} \quad \sum_{j=1}^{N} \vartheta_j = 1,$$

where ϑ_j is the expenditure share of sectoral consumption j and δ_c is a constant term.

Cost minimization derives the following demands for sectoral goods

$$C_{jt} = \frac{\vartheta_j C_t}{P_{jt}} \quad \text{for every } j,$$

where P_{jt} denotes the price of good j.

On the other hand, total hours are characterized by the following aggregator

$$L_t = \left(\sum_{j=1}^{N} L_{jt}^{\frac{1+\varrho}{\varrho}} \right)^{\frac{\varrho}{1+\varrho}},$$

where $\varrho \geq 0$ denotes the elasticity of substitution which governs the degree of labor mobility across sectors. This assumption follows Horvath (2000) by allowing imperfect labor mobility within a representative household framework.

The household maximizes total labor income, given sectoral wages and the labor aggregator. This problem derives the following labor supply schedule

$$L_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{\varrho} L_t \quad \text{for every } j,$$

with the aggregate wage index $W_t = \left(\sum_{j=1}^{N} W_{jt}^{1+\varrho}\right)^{\frac{1}{1+\varrho}}$.

Intertemporal decisions. On the intertemporal side, the household chooses sequences for consumption and domestic and foreign bonds, deriving the following optimality conditions

$$\begin{aligned}
1 &= \beta R_t \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1}{\Pi_{t+1}}\right] \\
1 &= \beta R_t^* \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{\Pi_{t+1}^e}{\Pi_{t+1}}\right] \\
W_t &= \kappa_t C_t^{\sigma} L_t^{\varphi},
\end{aligned}$$

where $\Pi_t = P_t/P_{t-1}$ and $\Pi_t^e = \mathcal{E}_t/\mathcal{E}_{t-1}$ denotes the gross inflation and the gross nominal depreciation rate, respectively. The first two conditions correspond to the Euler equations for domestic and foreign bonds, while the last condition corresponds to the aggregate labor supply.

2.2 Production

There are N different sectors in the economy indexed by j. Sector $j = 1$ corresponds to the commodity sector, while all other sectors correspond to other industries in the economy. We characterize each group in turn.

2.2.1 Domestic Non-commodity Sectors ($j \neq 1$)

Two layers of production characterize these sectors. At the top, there is a representative competitive firm aggregating the output of atomistic producers. The output of these firms is used to produce final goods that are used by the rest of the economy. At the bottom, there is a continuum of monopolistically competitive producers subject to nominal rigidities *à* la Calvo.

Sectoral aggregator. Each sectoral good j is produced by a competitive firm that aggregates the output of a continuum of intermediate producers according to

$$Y_{jt} = \left(\int_0^1 Y_{jt}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $z \in [0,1]$ denotes the z-th variety of good j and $\varepsilon > 0$ is the elasticity of substitution across varieties, which is common across sectors. Profit maximization derives the following demand faced by a producer z

$$Y_{jt}(z) = \left(\frac{P_{jt}(z)}{P_{jt}} \right)^{-\varepsilon} Y_{jt}, \tag{2}$$

where $P_{jt} = \left(\int_0^1 P_{jt}(z)^{1-\varepsilon} dz \right)^{\frac{1}{1-\varepsilon}}$ is the price of the final sectoral good j. The output of this sector is used either for consumption, exports, or as an input for other sectors of the economy.

Intermediate producers. Each producer $z \in [0,1]$ in sector j, operates in a monopolistically competitive environment. Technology is common to all firms within a sector but differs across sectors. The production function is given by

$$Y_{jt} = Z_{jt} \Big(\alpha_j^{\frac{1}{\sigma_Q}} L_{jt}^{\frac{\sigma_Q-1}{\sigma_Q}} + \nu_j^{\frac{1}{\sigma_Q}} V_{jt}^{\frac{\sigma_Q-1}{\sigma_Q}} + \mu_j^{\frac{1}{\sigma_Q}} M_{jt}^{\frac{\sigma_Q-1}{\sigma_Q}} \Big)^{\frac{\sigma_Q}{\sigma_Q-1}} \tag{3}$$

where L_{jt}, M_{jt}, and V_{jt}^m denote labor, domestic materials, and an importable good, respectively, and Z_{jt} is a sector-specific technology shock. $\sigma_Q \geq 0$ is the elasticity of substitution between production inputs, and $\alpha_j + \mu_j + \nu_j = 1$ holds in every sector.

The bundle of materials is sector-specific and given by

$$M_{jt} = \left(\sum_{i=1}^{N} \gamma_{ij}^{\frac{1}{\sigma_M}} M_{ijt}^{\frac{\sigma_M-1}{\sigma_M}} \right)^{\frac{\sigma_M}{\sigma_M-1}},$$

where M_{ijt} denotes sales of materials from sector i to sector j in period t. In the previous specification, the elasticity $\sigma_M \geq 0$ parameterizes the substitutability of different goods in the production of each industry's intermediate input bundle, while the elements $\gamma_{ij} \in [0,1]$ govern the production linkages between sectors in the economy, conditional on the level of usage of materials, μ_j.

Because technology and factor prices are the same across producers within a sector, the marginal cost is equal across monopolists, MC_{jt}. Given this, by cost minimization, the demands for labor, materials, and imports at the sectoral level are

$$L_{jt} = \alpha_j \left(\frac{W_{jt}}{MC_{jt}}\right)^{-\sigma_Q} \Delta_{jt} \frac{Y_{jt}}{Z_{jt}^{1-\sigma_Q}}$$
$$M_{jt} = \mu_j \left(\frac{P^m_{jt}}{MC_{jt}}\right)^{-\sigma_Q} \Delta_{jt} \frac{Y_{jt}}{Z_{jt}^{1-\sigma_Q}}$$
$$V^m_{jt} = \nu_j \left(\frac{P^m_{vt}}{MC_{jt}}\right)^{-\sigma_Q} \Delta_{jt} \frac{Y_{jt}}{Z_{jt}^{1-\sigma_Q}},$$

where $A_{jt} = \int_0^1 A_{jt}(z)dz$ for any $A = \{L, V, M\}$, $P^m_{jt} = \left(\sum_{i=1}^N \gamma_{ij} P_{it}^{1-\sigma_M}\right)^{\frac{1}{1-\sigma_M}}$ is the sector-specific price of intermediate goods, P^m_{vt} is the price of the importable good (common across sectors), and MC_{jt} denotes the average marginal cost of production in sector j. Also, by cost minimization, the demand for materials from sector i is given by $M_{ijt} = \gamma_{ij}(P_{it}/P^m_{jt})^{-\sigma_M} M_{jt}$. In the previous expressions, Δ_{jt} represents the sectoral price dispersion resulting from nominal rigidities and firms' pricing rules, which we describe next.

Nominal rigidities. Firms face price stickiness *à* la Calvo, implying that with probability $1-\theta_j$ they can reset their price in any given period, regardless of their last update. Therefore, a fraction θ_j of producers keep their price unchanged from the previous period, while the remaining fraction can set their desired price. This probability is sector-specific.

The desired price is chosen by maximizing profits subject to the demand of the sectoral aggregator, taking into account that this price can remain for an undetermined period of time, which is captured by the probability θ_j

$$\max_{\widetilde{P}_{jt}(z)} \sum_{k=0}^{\infty} \theta_j^k \mathbb{E}_t \Big\{ Q_{t,t+k} \Big[\widetilde{P}_{jt}(z) Y_{jt+k|t}(z) - MC_{jt+k} Y_{jt+k|t}(z) \Big] \Big\} \quad \text{subject to Eq. (2)},$$

where $\widetilde{P}_{jt}$ denotes the desired price and $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{1}{\Pi_{t+k}}$ is the stochastic discount factor of the household. In the previous expression, $Y_{jt+k|t}(z)$ denotes the demand of the sectoral aggregator in period $t+k$, conditional on the last price reset of firm z in period t.

The optimality condition is

$$\widetilde{P}_{jt} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{\sum_{k=0}^{\infty} (\theta_j \beta)^k \mathbb{E}_t \left(\frac{C_{t+k}^{-\sigma}}{P_{t+k}} MC_{t+k} P^{\varepsilon}_{jt+k} Y_{jt+k} \right)}{\sum_{k=0}^{\infty} (\theta_j \beta)^k \mathbb{E}_t \left(\frac{C_{t+k}^{-\sigma}}{P_{t+k}} P^{\varepsilon}_{jt+k} Y_{jt+k} \right)} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{S_{jt}}{F_{jt}},$$

where S_{jt} aggregates discounted nominal marginal costs under Calvo survival and F_{jt} aggregates discounted demands. Those terms can be written recursively as

$$S_{jt} = C_t^{-\sigma} Y_{jt} MC_{jt} + \theta_j \beta \mathbb{E}_t[\Pi_{jt+1}^{\varepsilon} S_{jt+1}]$$

$$F_{jt} = C_t^{-\sigma} Y_{jt} + \theta_j \beta \mathbb{E}_t[\Pi_{t+1}^{-1} \Pi_{jt+1}^{\varepsilon} F_{jt+1}]$$

where $\Pi_{jt} = P_{jt}/P_{jt-1}$ is the sectoral inflation.

Given the properties of the Calvo pricing setup, sectoral prices and the sectoral price dispersion evolve as

$$1 = (1-\theta_j)\left(\frac{\widetilde{P}_{jt}}{P_{jt}}\right)^{1-\varepsilon} + \theta_j \Pi_{jt}^{\varepsilon-1}$$

$$\Delta_{jt} = (1-\theta_j)\left(\frac{\widetilde{P}_{jt}}{P_{jt}}\right)^{-\varepsilon} + \theta_j \Pi_{jt}^{\varepsilon} \Delta_{jt-1}.$$

2.2.2 Commodity Sector ($j = 1$)

The commodity sector is composed of a representative competitive firm with the same technology as other domestic producers. However, there are two main differences. First, given its competitive nature, this sector takes the commodity price in foreign markets as given, which follows an exogenous AR(1) process. Second, this sector is not subject to nominal rigidities for its domestic price, and the law of one price holds. These assumptions imply that $P_{1t} = MC_{1t} = \mathcal{E}_t P_{1t}^*$ where the process of the foreign price is given by

$$\log(P_{1t}^*/P_t^*) = (1-\rho_1)\log(p_1^*) + \rho_1 \log(P_{1t-1}^*/P_{t-1}^*) + \upsilon_t, \qquad \upsilon_t \sim N(0, \sigma_1).$$

2.3 Exportable Good

There is a competitive firm that combines inputs from different domestic non-commodity sectors to produce an exportable good, which is fully sold abroad. The technology of this sector is given by $Y_t^x = \delta_x \prod_{j=2}^{N} (Y_{jt}^x)^{\psi_j}$, where δ_x is a constant term and $\sum_{j=2}^{N} \psi_j = 1$ holds. The price of exports and the demand for inputs from each sector are given by

$$P_t^x = \prod_{j=2}^{N} P_{jt}^{\psi_j} \qquad \text{and} \qquad Y_{jt}^x = \psi_j \frac{P_t^x}{P_{jt}} Y_t^x.$$

The foreign demand for the exportable good takes the form $Y_t^x = \omega^x \left(\frac{P_t^x}{\mathcal{E}_t P_t^*}\right)^{-\eta^*} Y_t^*$, where P_t^* and Y_t^* denote the foreign CPI and the foreign level of output, respectively.

2.4 Importable Good

In the model, sectoral producers are the only agents who import goods abroad.[3] Therefore, external prices and the real exchange rate play a major role in determining the cost of production of firms. It is standard to assume that the law of one price holds for foreign goods, especially in models analyzing the role of terms of trade shocks (Dib, 2008; Caputo and Irarrazaval, 2017; Drechsel et al., 2019; Cao and Dong, 2020). However, plenty of evidence indicates that the law of price holds only "at the dock" for imported goods in the short run, so there is a partial exchange rate pass-through to the rest of the economy at business cycle frequencies. For example, the seminal work of Campa and Goldberg (2005) documents that the exchange rate pass-through to imported prices is only 46% across a sample of OECD countries in the period 1975-2003. More recently, Brun-Aguerre et al. (2012) and Choudhri and Hakura (2015) provide similar results for a sample of developed and emerging countries. On the other hand, Gopinath and Rigobon (2008), and Gopinath et al. (2010) provide additional evidence for the U.S. and Campa and Gonzalez Minguez (2006) for the Euro Area, respectivelly.[4]

To model limited exchange rate pass-through, we incorporate monopolistic competition and nominal rigidities in the importable good sector (Smets and Wouters, 2002; Monacelli, 2005; Garcia-Schmidt and Garcia-Cicco, 2020).[5] As we will see later in the theoretical and quantitative analysis, the interaction of nominal rigidities across sectors, mediated by production linkages, will generate rich interactions in the determination of aggregate variables.

There exists an importable good sector in the economy with similar features to other sectors. There is a continuum of monopolistically competitive firms that demand imports from abroad and transform these goods into differentiated varieties. The output of these firms is combined by a perfectly competitive firm in a CES fashion and sold to domestic sectors for production

$$V_t^m = \left(\int_0^1 V_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \text{with demand} \qquad V_t(z) = \left(\frac{P_{vt}(z)}{P_{vt}^m} \right)^{-\varepsilon} V_t^m,$$

[3]This assumption is without loss of generality. A model in which the representative household also imports would be isomorphic to a version in which (i) one sector only produces with imports, and (ii) this sector sells to the household, with an appropriate weight ϑ capturing the degree of home bias.

[4]Recent additional evidence for developed and emerging economies on exchange rate pass-through to other prices can be found, among many others, in Choudhri and Hakura (2006) and Kohlscheen (2010). See Burstein and Gopinath (2014) for a survey.

[5]An alternative approach to model the limited exchange rate pass-through is to consider local distribution costs. See for example Devereux and Engel (2002), Corsetti and Dedola (2005) and Atkeson and Burstein (2008).

where $P^m_{vt} = \left(\int_0^1 P_{vt}(z)^{1-\varepsilon} dz\right)^{\frac{1}{1-\varepsilon}}$ is the price paid by domestic producers for importable inputs. Total output of this sector must be equal to the amount demanded by other sectors for production, $V^m_t = \sum_j V_{jt}$. For simplicity, we assume that the elasticity of substitution among varieties is the same as in other domestic sectors.

The monopolistically competitive firms $z \in [0, 1]$ import a homogeneous good from abroad and transform it into differentiated varieties using a linear technology, $V_t(z) = V^*_t(z)$. These firms are subject to nominal rigidities *à* la Calvo, with a probability of price adjustment given by $1 - \theta_v$. The optimization problem is to maximize profits subject to demand by choosing the optimal price $\widetilde{P}_{vt}(z)$. This problem is similar to the one presented before for other sectors. Letting P^*_{vt} be the foreign price of imports and assuming that the law of one price holds, the domestic price of this good is equal to the nominal exchange rate times the foreign price, $P_{vt} = \mathcal{E}_t P^*_{vt}$. Moreover, assuming that the price of the importable good is the same as the foreign CPI ($P^*_{vt} = P^*_t$), the marginal cost of production in the importable sector, in units of the domestic consumption good, corresponds to the real exchange rate. From now on, we work directly with the real exchange rate, q_t, as the marginal cost of importable goods, rather than P_{vt}.

Due to nominal rigidities, there is a price dispersion across importable goods, which is denoted by $\Delta_{vt} \geq 1$. This implies that the quantity sold to domestic producers (V^m_t) is lower than the quantity imported from abroad (V_t), so $V_t = \Delta_{vt} V^m_t$ holds. The value added of the importable good sector is given by $VA_{vt} = P_{vt} V^m_t - \mathcal{E}_t P^*_t V_t$ and is rebated as profits to the household.

2.5 Aggregation, Monetary Policy and Market Clearing

Total exports (in nominal terms) are given by the value of commodity goods plus the value of the exportable good, $X_t = P_{1t} Y^x_{1t} + P^x_t Y^x_t$. Therefore, the nominal border-priced trade balance is given by $TB_t = X_t - \mathcal{E}_t P^*_t V_t$.

Nominal GDP (Y^n_t) is defined as the sum of the value added of domestic producers j and the value added of the importable good sector. Using the market-clearing conditions of each sector, this can be expressed as $Y^n_t = P_t C_t + TB_t$. In real terms (in units of the consumption good), real GDP is $Y_t = C_t + \frac{TB_t}{P_t}$.

The domestic nominal rate is set by the central bank using a Taylor rule

$$\frac{R_t}{R} = (\Pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_y},$$

where R is the long-run value of the gross nominal rate.

For equilibrium, we impose market-clearing for every sectoral good

$$Y_{jt} = C_{jt} + Y_{jt}^{x} + \sum_{i=1}^{N} M_{jit}.$$

By combining the different market-clearing conditions of the model, the domestic bond market clearing ($B_t = 0$) and the aggregate resource constraint (given by the budget constraints of the household), the financial position of the economy is given by

$$\mathcal{E}_t B_t^* = TB_t + \mathcal{E}_t R_{t-1}^* B_{t-1}^*.$$

To close the model, we consider the following specification for the foreign interest rate

$$R_t^* = R_t^w \times \exp\left[\phi_b \left(\bar{b} - \frac{\mathcal{E}_t B_t^*}{Y_t^n}\right)\right],$$

where $\phi_b > 0$ and R_t^w is the world interest rate, and the term $\exp[\cdot]$ is a risk premium that the country pays over the risk-free rate. This premium is increasing in the deviations of the country's total debt from its long-run value, normalized by GDP. This device guarantees the stationarity of the model (Schmitt-Grohe and Uribe, 2003).

3 Theoretical Results

In this section, we examine a simplified version of the model to investigate how nominal rigidities and input-output linkages influence the transmission of the commodity price shock. We focus on the impact-response of the economy to the shock and assume (i) an economy in financial autarky, implying trade balance at every period, (ii) the commodity good is an endowment normalized to one and fully exported abroad, (iii) Cobb-Douglas technologies ($\sigma_Q = \sigma_M = 1$), and (iv) the economy is in steady-state in period $t-1$ and its expected to come back to steady state from period $t+1$ onwards. This latter assumption implies that our analysis is focused on the impact response to the commodity price shock and that dynamics play no role.

In what follows, we construct the equilibrium of the small open economy in steps. Lower case nominal variables are deflated by aggregate CPI, and hat variables represent log-linear deviations with respect to the steady-state (i.e., $\widehat{x}_t = \log(x_t) - \log(x)$).[6]

[6]With Cobb-Douglas technologies, the approximation of production functions holds exactly in logs; we use hats nota-

3.1 Equilibrium in the Simplied Economy

The real exchange rate. An approximation of the trade balance reads as

$$\widehat{tb}_t = \left(\frac{p_1 Y_1^x}{x}\right)(\widehat{p}_{1t} + \widehat{Y}_1^x) + \left(\frac{p^x Y^x}{x}\right)(\widehat{p}_t^x + \widehat{Y}_t^x) - \left(\frac{qV}{x}\right)(\widehat{q}_t + \widehat{V}_t) = 0.$$

Without loss of generality and to simplify the algebra, we assume $\widehat{V}_t = \widehat{Y}_t^* = \widehat{p}_t^x = 0$. Letting $s_x = p_1 Y_1^x / x$ and $1 - s_x = p^x Y^x / x$ and using the law of one price for commodities ($\widehat{p}_{1t} = \widehat{q}_t + \widehat{p}_{1t}^*$), the previous expression pins down a closed-form solution for the real exchange rate

$$\widehat{q}_t = -\left(\frac{s_x}{(1 - s_x)(\eta^* - 1)}\right)\widehat{p}_{1t}^* = \kappa_q \widehat{p}_{1t}^*.$$

Under the empirically relevant case $\eta^* > 1$ and because $s_x \in (0, 1)$, the term in parentheses is positive and $\kappa_q < 0$ holds. Therefore, a positive commodity price shock generates an exchange rate appreciation. Intuitively, as the trade balance is equal to zero, any commodity price windfall must be offset by a real appreciation that reduces non-commodity exports. Because $\eta^* > 1$, foreign demand for the export composite is elastic, and the appreciation cuts volumes enough. In this case, the mechanism operates through relative price movements, rather than through net exports.

Sectoral prices. All prices subject to nominal rigidities (for domestic non-commodity industries $z = j$ and for the importable good sector $z = v$) are characterized by a log-linearized New Keynesian Phillips curve (NKPC) of the form

$$\pi_{zt} = \beta \mathbb{E}_t[\pi_{zt+1}] + \kappa_z \widehat{mc}_{zt} \quad \text{with } \kappa_z = \frac{(1 - \theta_z)(1 - \beta\theta_z)}{\theta_z}.$$

Noting that sectoral inflation can be written as $\pi_{zt} = \widehat{p}_{zt} - \widehat{p}_{zt-1} + \pi_t$ and using our assumptions, the NKPC reads as $\widehat{p}_{zt} + \pi_t = \kappa_z \widehat{mc}_{zt}$. Therefore, at the margin, $\widehat{p}_{zt} \approx \widetilde{\kappa}_z \widehat{mc}_{zt}$ with $\widetilde{\kappa}_z = (1 - \theta_z)(1 - \beta\theta_z)$. The latter expression implies that lower price rigidities increase the pass-through from marginal costs to prices, as $\widetilde{\kappa}_z$ is strictly decreasing in θ_z ($\partial\widetilde{\kappa}_z / \partial\theta_z < 0$) and bounded in $[0, 1]$. For the rest of this section, we refer to price flexibility either as a decrease in θ_z or as an increase in $\widetilde{\kappa}_z$.

Using the previous derivations, for the importable sector ($z = v$), the price of imported materials reads as $\widehat{p}_{vt} = \widetilde{\kappa}_v \widehat{q}_t = \widetilde{\kappa}_v \kappa_q \widehat{p}_{1t}^*$, because its marginal cost equals the real exchange rate. For domestic

tion for expositional consistency. Also note that this assumption is made without loss of generality, as CES technologies only change the weights associated with the approximations, not the qualitative insights.

sectors ($z = j \neq 1$), the marginal cost takes the form $\widehat{mc}_{jt} = \alpha_j \widehat{w}_{jt} + \nu_j \widehat{p}_{vt} + \mu_j \sum_{i \neq 1} \gamma_{ij} \widehat{p}_{it}$. Combining with the expression for sectoral prices, we get

$$\widehat{p}_{jt} = \widetilde{\kappa}_j \left(\alpha_j \widehat{w}_{jt} + \nu_j \widehat{p}_{vt} + \mu_j \sum_{i \neq 1} \gamma_{ij} \widehat{p}_{it} \right).$$

Assuming a high degree of labor mobility ($\varrho \to \infty$), sectoral wages are equalized ($w_{jt} = w_t$). At the same time, there is a positive association between the commodity price shock and wages, as the former represents a windfall for the economy. We characterize this relation in reduced form as $\widehat{w}_t = \kappa_w \widehat{p}^*_{1t}$ with $\kappa_w > 0$.[7]

Our previous results imply that domestic sectoral prices read as

$$\widehat{p}_{jt} = \widetilde{\kappa}_j \left(\alpha_j \kappa_w \widehat{p}^*_{1t} + \nu_j \widetilde{\kappa}_v \kappa_q \widehat{p}^*_{1t} + \mu_j \sum_{i \neq 1} \gamma_{ij} \widehat{p}_{it} \right).$$

We now characterize these prices in closed form. First, let $\mathbf{p}_t$ be the vector of domestic sectoral prices other than commodities, with characteristic element $\widehat{p}_{jt}$. Also, collect all the relevant parameters in the following vectors: $\boldsymbol{\alpha} = (\alpha_j)$, $\boldsymbol{\nu} = (\nu_j)$, $\boldsymbol{\vartheta} = (\vartheta_j)$, $\widetilde{\boldsymbol{\kappa}} = (\widetilde{\kappa}_j)$, $\boldsymbol{\mu} = (\mu)$ and the input-output (IO) matrix $\boldsymbol{\Gamma}$ with nonnegative terms γ_{ij}. Defining the auxiliary arrays $\mathbf{B} = \mathrm{diag}(\widetilde{\boldsymbol{\kappa}})\, \mathrm{diag}(\boldsymbol{\mu}) \boldsymbol{\Gamma}'$, $\mathbf{u} = (\boldsymbol{\alpha} \kappa_w + \boldsymbol{\nu} \widetilde{\kappa}_v \kappa_q)$, $\mathbf{a} = \mathrm{diag}(\widetilde{\boldsymbol{\kappa}}) \mathbf{u}$ and the Leontief inverse $\mathbf{H} = (\mathbf{I} - \mathbf{B})^{-1}$, prices are given by

$$\mathbf{p}_t = (\mathbf{I} - \mathbf{B})^{-1} \mathbf{a} \widehat{p}^*_{1t} = \mathbf{H} \mathbf{a} \widehat{p}^*_{1t}.$$

At this point, note that sectoral prices transmit the commodity shock through two channels. First, the commodity boom is an income windfall that raises wages and pushes costs up, an effect governed by $\boldsymbol{\alpha} \kappa_w$, which we refer to as the wage-push channel. Second, a positive commodity price appreciates the real exchange rate, making imports cheaper in domestic currency and thereby reducing costs. This is the import-cost relief channel. How important this channel is depends on $\boldsymbol{\nu} \widetilde{\kappa}_v \kappa_q$, so nominal rigidities for imported goods are key to mediating the strength of this force. Also note that price changes in one sector propagate to others via IO multipliers captured by $\mu_j \gamma_{ij}$ in the Leontief inverse.

On the other hand, we observe that nominal rigidities in sector j have two effects. First, they dampen the pass-through from costs to prices, which is captured by vector $\mathbf{a}$. Second, they appear inside the IO multiplier matrix, so the whole network amplification is attenuated when prices are sticky. These features are common to the closed economy case studied by Pasten et al. (2020).

[7] Even though we do not provide microfoundations for this relation, in our quantitative analysis, we verify that this condition holds. Romero (2025) provides empirical evidence of this result.

Inflation and GDP. From the CPI aggregation, $\pi_t = \sum_{j\neq 1} \vartheta_j \pi_{jt}$ holds. Aggregating across sectors we have $\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \sum_{j\neq 1} \vartheta_j \widetilde{\kappa}_j \widehat{mc}_{jt} = \sum_{j\neq 1} \vartheta_j \widehat{p}_{jt}$, which can be written as

$$\pi_t = \boldsymbol{\vartheta}'(\mathbf{I}-\mathbf{B})^{-1}\mathbf{a}\widehat{p}^*_{1t}. \tag{4}$$

As aggregate inflation is a weighted average of domestic sectoral prices, filtered by IO linkages and stickiness, the sign and size depend on whether the wage-push channel dominates the import-cost relief channel. With more rigid import prices (low pass-through), the appreciation does not reduce costs quickly enough, resulting in higher inflation. The opposite happens with flexible import prices, where disinflation is stronger. This is one of the main differences with respect to Pasten et al. (2020) and our key contribution to the literature: the source of nominal rigidities (in domestic producers versus in the import sector) reverses the effect on inflation. We discuss this result in more detail below.

From the assumption of financial autarky and the definition of GDP, $\widehat{Y}_t = \widehat{C}_t$ holds. Using the Euler equation for domestic bonds and the Taylor rule, we get $\widehat{Y}_t = -\frac{\phi_\pi}{\sigma+\phi_y}\pi_t$, implying that all mechanisms affecting inflation in one direction do it in the opposite direction for GDP, mediated by the monetary policy response to inflation and output fluctuations.

3.2 The Role of Nominal Rigidities

We now utilize the simplified framework to investigate the role of key parameters in determining equilibrium. We start by analyzing the role of nominal rigidities in the importable goods sector. To save space, we relegate the proofs to Appendix A.

Proposition 1. *An increase in import price flexibility reduces inflation,* $\frac{\partial \pi_t}{\partial \widetilde{\kappa}_v} < 0$.

In other words, faster import pass-through allows the appreciation to hit input costs more quickly, thereby lowering inflation on impact. Note that the strength of this channel depends on the degree of import usage (ν) and how the appreciation propagates from marginal costs to prices ($\widetilde{\kappa}$).

The following proposition summarizes the implications of price flexibility for domestic non-commodity producers.

Proposition 2. *The impact of an increase in price flexibility in any domestic sector j reads as*

$$\frac{\partial \pi_t}{\partial \widetilde{\kappa}_j} = \Big(\underbrace{\boldsymbol{\vartheta}'\mathbf{H}\mathbf{E}_j\mathbf{u}}_{\textit{direct}} + \underbrace{\boldsymbol{\vartheta}'\mathbf{H}\mathbf{E}_j \operatorname{diag}(\boldsymbol{\mu})\boldsymbol{\Gamma}'\mathbf{H}\mathbf{a}}_{\textit{network-multiplier}}\Big)\widehat{p}^*_{1t}$$

where $\mathbf{E}_j$ is the selection matrix with a 1 at element (j,j) and zeros elsewhere. Moreover, if the sectorwise dominance condition $u_j = \alpha_j \kappa_w + \nu_j \widetilde{\kappa}_v \kappa_q \geq 0$ holds for every j, then $\frac{\partial \pi_t}{\partial \widetilde{\kappa}_j} \geq 0$.

Proposition 2 states that more domestic price flexibility raises each sector's own pass-through (direct effect) and strengthens the network multiplier $\mathbf{H} = (\mathbf{I} - \mathbf{B})^{-1}$ (network-multiplier effect), so wage-push diffuses faster and further. If import-cost relief does not outweigh the wage-push effect (i.e., the sectorwise dominance condition is satisfied), inflation rises with domestic price flexibility.

Taking stock, while the wage-push channel generates inflationary pressures, the import-cost relief channel goes in the opposite direction. Therefore, the response of inflation depends on the relative strength of each force, which critically depends on the relative nominal rigidities between imports and domestic sectors. The following corollaries formalize two knife-edge cases derived from our propositions.

Corollary 1. *With rigid prices in the domestic sectors j and flexible prices in the importable sector, the impact-response of inflation is small and could be negative.*

This scenario corresponds to the case where $\widetilde{\kappa}_j \to 0$ for all j and $\widetilde{\kappa}_v \to 1$. This implies that $\mathbf{B} \to \mathbf{0}$ and $\mathbf{H} \to \mathbf{I}$ and the network multiplier effect is nil. Moreover, aggregate inflation reads as $\pi_t = \sum_j \vartheta_j \widetilde{\kappa}_j (\alpha_j \kappa_w + \nu_j \kappa_q) \widehat{p}^*_{1t}$ which, due to $\widetilde{\kappa}_j \to 0$, has a small magnitude and might be negative. Intuitively, domestic prices barely move, so there is no network amplification. Imports reprice immediately, so the appreciation of the currency makes inputs cheaper, but this disinflation cannot propagate much. The impact over inflation is small and its sign depends on the balance between $\alpha_j \kappa_w$ and $\nu_j |\kappa_q|$.

Corollary 2. *With flexible prices in the domestic sectors j and rigid prices in the importable sector, the impact-response of inflation is positive.*

This opposite case happens when $\widetilde{\kappa}_j \to 1$ for all j and $\widetilde{\kappa}_v \to 0$, implying an impact response of inflation given by $\pi_t = \boldsymbol{\vartheta}'(\mathbf{I} - \mathrm{diag}(\boldsymbol{\mu})\boldsymbol{\Gamma}')\boldsymbol{\alpha}\kappa_w \widehat{p}^*_{1t} > 0$. In this case, wage-push is fully transmitted and amplified by the IO network, while import-cost relief is shut off because imports are sticky. As a result, inflation rises on impact.

Our previous results focus on the behavior of inflation. Our following proposition presents results for the response of output.

Proposition 3. *An increase in the price flexibility of imports (sector j) increases (decreases) output.*

This is a direct result of the inverse relationship between inflation and output resulting from monetary policy in the autarkic economy. Anything that lowers inflation on impact (i.e., more flexible imports) makes the policy reaction milder and supports output. In contrast, anything that raises inflation (i.e., more flexible domestic prices) tightens policy and reduces output.

In summary, the results in Propositions 1 and 2 indicate a tension between the two channels associated with nominal rigidities, the wage-push and the import-cost relief channels, resulting in ambiguous predictions about the behavior of aggregate inflation following a commodity price shock. There exists a threshold for the degree of nominal rigidities in imports such that the wage-push channel dominates the import-cost relief channel and the economy experiences inflationary pressures. As the response of output is positive but inversely proportional to import nominal rigidities (Proposition 3), the behavior of the response might look like the response to either a positive aggregate demand shock or aggregate supply shock. This feature is, by construction, not present in models of closed economies, such as Pasten et al. (2020). To the best of our knowledge, it is a novel contribution to the literature.

3.3 The Role of Production Linkages

The previous section analyzed the role of nominal rigidities in shaping the response of inflation and output. Now we focus on the role played by production linkages. First, we analyze the impact of changes in material usage for production (changes in μ_j in the production function). We assume a proportional change in the usage of labor and imports, so constant returns to scale hold, and focus only on the response of inflation.

Changes in the intensity of usage of materials. The derivative of inflation with respect to the usage of materials is

$$\begin{aligned}\frac{\partial \pi_t}{\partial \mu_j} &= \boldsymbol{\vartheta}'\left(\frac{\partial \mathbf{H}}{\partial \mu_j}\mathbf{a} + \mathbf{H}\frac{\partial \mathbf{a}}{\partial \mu_j}\right)\widehat{p}^*_{1t} \\ &= \boldsymbol{\vartheta}'\Big(\mathbf{H}\big(\operatorname{diag}(\widetilde{\boldsymbol{\kappa}})\mathbf{E}_j\boldsymbol{\Gamma}'\big)\mathbf{H} - (\phi_{\alpha_j}\kappa_w + \phi_{\nu_j}\widetilde{\kappa}_v\kappa_q)\mathbf{H}\operatorname{diag}(\widetilde{\boldsymbol{\kappa}})\mathbf{e}_j\Big)\widehat{p}^*_{1t},\end{aligned}$$

where $\mathbf{e}_j$ is a selection vector with a one at element j and zeros elsewhere, and where ϕ_{α_j} and ϕ_{ν_j} capture the adjustment strength in the usage of labor and imports (in absolute value), respectively.

In principle, the sign of the previous expression is ambiguous. On the one hand, raising μ_j increases propagation through the IO network, which corresponds to the first term in parentheses.

However, stronger usage of materials might reduce the need for labor and imported goods to maintain constant returns to scale, thereby reducing both the wage-push and import-relief effects. If there is no reallocation of inputs, the latter channel is muted, and stronger usage of materials leads to inflationary pressures. Therefore, the conclusions of the comparative static behind this exercise depend on the productive structure of the counterfactual economy.

Changes in the intensity of IO linkages. In the same way, the effect of production linkages also comes from the intensity of connections between sectors, which is captured by γ_{ij}. Fixing a customer sector j, the impact of an increase in the usage of materials provided by sector i is

$$\frac{\partial \pi_t}{\partial \gamma_{ij}} = \boldsymbol{\vartheta}' \Big(\mathbf{H} \operatorname{diag}(\widetilde{\boldsymbol{\kappa}}) \operatorname{diag}(\boldsymbol{\mu}) \big(\mathbf{E}_{ji} - \sum_{k \neq i} \mathbf{E}_{jk} \big) \mathbf{H} \mathbf{a} \Big) \widehat{p}^*_{1t}.$$

Similar to the case where there is a change in the usage of materials, the response of inflation to a change in the intensity of linkages is ambiguous. There would be inflationary pressures whenever there is a reweight toward suppliers where the wage-push channel is stronger than the import-relief channel (positive $\mathbf{E}_{ji} - \sum_{k \neq i} \mathbf{E}_{jk}$ term for sectors where $\mathbf{a}$ is positive).

4 Quantitative Analysis

This section explores the quantitative properties of the model. We start by describing the calibration strategy applied to Chile. We then analyze the impact of commodity price shocks under various configurations of nominal rigidities and production linkages. Finally, we conduct robustness checks and examine the implications of our mechanisms under alternative driving forces.

4.1 Calibration

The frequency of the model is one quarter, and the number of sectors is $N = 3$, replicating an economy characterized by commodities, manufactured goods ($j = m$), and services ($j = s$). To calibrate the economy, we partition the parameter space into two groups. The first subset of parameters is taken from the literature or calibrated to match long-run features of the data. We select the second group of parameters governing the model's dynamics to match the empirical responses to a commodity price shock. Table B.1 summarizes the calibration of the model.

The parameter for the risk aversion is equal to one, and the (inverse of the) Frisch elasticity is 1.85 (Chetty, 2011). We calibrate the discount factor to obtain a domestic interest rate of 5.8 percent at an annual frequency, in line with Garcia-Schmidt and Garcia-Cicco (2020). The labor supply elasticity is set to 1, implying limited labor mobility across sectors.

For productive sectors, the elasticity of substitution across varieties is set to 10, implying a steady-state level of sectoral profits of approximately 11 percent, which is among the values in the New Keynesian literature. Following Atalay (2017), we set $\varepsilon_Q = 1$ and $\varepsilon_M = 0.1$, implying a Cobb-Douglas structure for gross output and complementarities for the bundle of materials. We use the Input-Output (IO) table for Chile of 2017 to fix the parameters associated with the production functions (α_j, ν_j, μ_j) and match the empirical expenditure shares to the model. We aggregate the 111 sectors in the IO table to build an aggregate for the commodity (corresponding to mining activities), manufacturing, and services sectors. Services are more labor-intensive than the rest of the sectors ($\alpha_s = 0.48$), while commodity and manufacturing have a similar labor intensity ($\alpha_c = 0.19$ and $\alpha_m = 0.24$). On the other hand, the three sectors intensively use domestic materials ($\mu_c = 0.69$, $\mu_m = 0.60$, and $\mu_s = 0.45$), leaving a usage of imports that varies between 8 percent for services and 17 percent for manufactures. The intensity of specific materials (input-output) usage also comes from the IO (2017) and fluctuates between 0.45 and 0.65. Finally, the manufacturing sector represents 83 percent of the composition of the exportable good, pinning down parameters ψ_j of the exportable good aggregator.

To calibrate the level of price rigidities, we follow Pasten et al. (2020) and compute the average frequency of price adjustment for goods in the different categories. For this, we rely on the microdata underlying the construction of the official Chilean CPI for the period 2010-2018 and compute the ratio between the number of price changes over the total number of periods available for each product. Then, we aggregate those individual products to get the behavior of manufactures and services. Then, the Calvo parameters correspond to one minus the obtained level. With this procedure, we obtain values of $\theta_m = 0.46$ and $\theta_s = 0.59$, implying that the price of manufactures is relatively more flexible than the price of services. For the degree of nominal rigidities in the importable sector, we set a baseline value of $\theta_v = 0.52$ to match the exchange rate pass-through to import prices of 0.46 reported by Campa and Goldberg (2005). However, given the relevance of nominal rigidities in the importable sector, we provide extensive robustness exercises throughout this section.

The values for the monetary policy rule are standard in the literature, with a central bank reacting aggressively to movements in inflation ($\phi_\pi = 1.5$) and more moderately to output fluctuations ($\phi_y = 0.125$). The world interest is taken from Garcia-Schmidt and Garcia-Cicco (2020) and set to 4.5

percent in annual terms. We set the long-run value of foreign bonds to match a trade balance-to-GDP ratio of 8 percent in the steady state. For the foreign demand, we normalize $\omega^x = 1$ and set $\eta^* = 1.4$ following Adolfson et al. (2007).

The final set of parameters corresponds to the persistence of the commodity price and the sensitivity of the interest rate premium to fluctuations in debt. To calibrate them, first, we run a structural VAR (SVAR) model at a quarterly frequency between 1996 and 2018, including (i) the foreign price of copper (the nominal price in dollars divided by the U.S. CPI); (ii) consumption; (iii) output; (iv) trade balance; and (v) the real exchange rate. All variables are in logs, except the trade balance (as a fraction of total output). The model includes a linear and a quadratic trend as exogenous controls. The identifying assumption in the SVAR model is that Chile is a small open economy, so the commodity price does not react to domestic conditions. For parsimony, we estimate the model with only one lag. Then, we set the parameters' values to match the theoretical and empirical responses of consumption, the trade balance, and the real exchange rate to the commodity price shock.[8] This procedure gives a value of $\phi_b = 0.009$ for the interest rate premium, and $\rho_1 = 0.87$ and $\sigma_1 = 0.013$ for the autoregressive parameter of the commodity price. Fig. B.1 in the appendix compares the impulse-response function of the model with the data for selected aggregate variables, demonstrating that it replicates the main features of a commodity price shock.

4.2 The Role of Nominal Rigidities

We begin the analysis by examining the role of nominal rigidities in transmitting commodity price shocks. Fig. 1 compares the impact response of aggregate variables to a one standard deviation increase in the commodity price under the baseline scenario with nominal rigidities (solid blue line) vs a scenario with flexible prices (dashed red line) for domestic non-commodity sectors. In all figures, the horizontal axis denotes the degree of nominal rigidities in importable goods, moving from fully flexible ($\theta_v = 0$) to fully rigid ($\theta_v = 1$).

Panel a presents the impact response of inflation. There are two observations from the figure. First, the threshold in the value of nominal import rigidities, such that the shock shifts from deflationary to inflationary, is close to 0.2. To the right of this threshold, the wage-push effect

[8]Formally, this procedure corresponds to a Simulated Method of Moments (SMM), in which the vector of parameters solves $\arg\min_{\theta\in\Theta}(\widehat{\gamma}_T - f(\theta))' W_T(\widehat{\gamma}_T - f(\theta))$. In this expression, $\widehat{\gamma}_T$ is the structural impulse-response function obtained from a sample of dimension T, θ corresponds to the structural parameters of the model, $f(\cdot)$ defines the theoretical response, and W_T is a weighting matrix given by the inverse of the interquartile range of the empirical impulse-response.

dominates over the import-cost relief effect as the exchange rate appreciation is muted, consistent with the results in Proposition 1. Note that our baseline calibration for nominal rigidities is well to the right of this threshold, so the baseline calibration generates inflation after the shock. Second, the magnitude of the impact response of inflation depends on the degree of nominal rigidities in other domestic sectors. Higher price flexibility in domestic sectors amplifies the propagation of the commodity price shock through a higher price pass-through from prices to marginal costs and the network multiplier, in line with Proposition 2.

As the increase in commodity prices represents a windfall for the economy, there is an increase in output and consumption, which is exacerbated when there are nominal rigidities in domestic sectors, as shown in panels b and d. This "inverted" result is mediated by the response of monetary policy in panel c, which adjusts more strongly with nominal rigidities in every sector, conditional on a specific Taylor rule. This is because, even though inflation is lower when there are nominal rigidities, the stronger response of output generates a more aggressive response of policy, in line with Proposition 3.

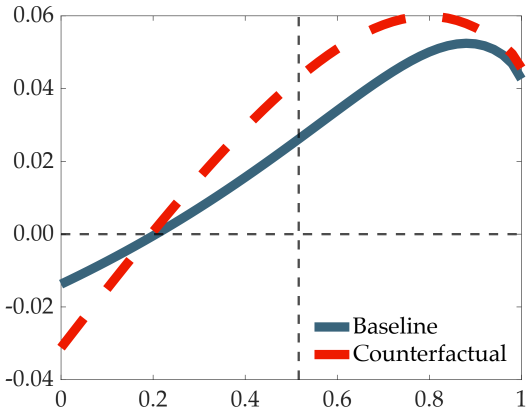
Combining the response of output, which is always positive, with the ambiguous response of inflation, we show that the (positive) commodity price shock can be interpreted either as an expansionary demand shock or an expansionary supply shock. This novel result in the literature has significant implications for the conduct of monetary policy, particularly in relation to the degree of nominal rigidities in the importable sector.

Finally, panel e shows that more nominal rigidities in imports imply a more negative response in the trade balance, except for very high degrees of nominal rigidities. In this case, the exchange rate operates less as an adjustment mechanism, and quantities react more. However, there are no relevant differences between the baseline case and the counterfactual one with flexible domestic prices. This is not the case for the real exchange rate in panel f, which appreciates more whenever there are more flexible prices in domestic sectors.

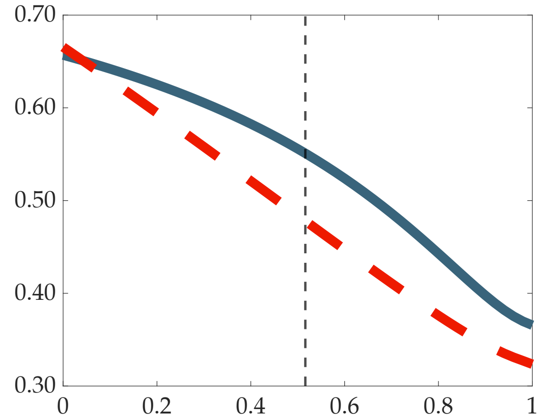
4.3 The Role of Production Linkages

We now examine the interaction between nominal rigidities in the importable sector and the degree of domestic material usage in production. For this, we compare our baseline economy with one in which we decrease/increase the usage of materials by 10%, which reflects a decrease/increase in parameter μ_j in Eq. (3), accompanied by a proportional rise in parameters α_j and ν_j. As in the previous section, we focus on the impact response of the economy.

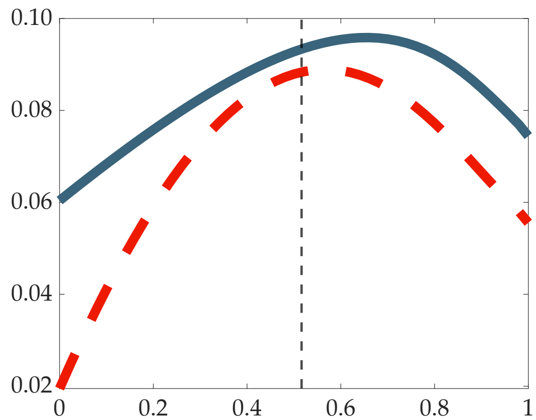
Figure 1. The Role of Nominal Rigidities



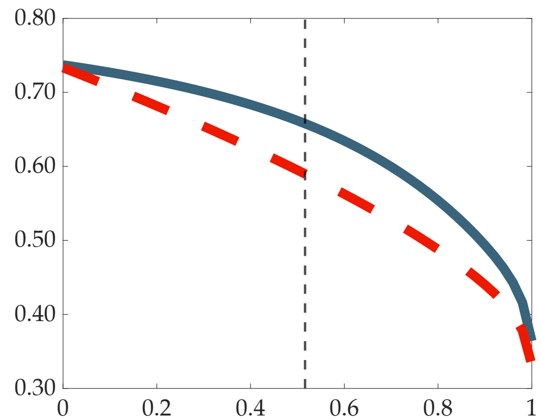
(a) Inflation



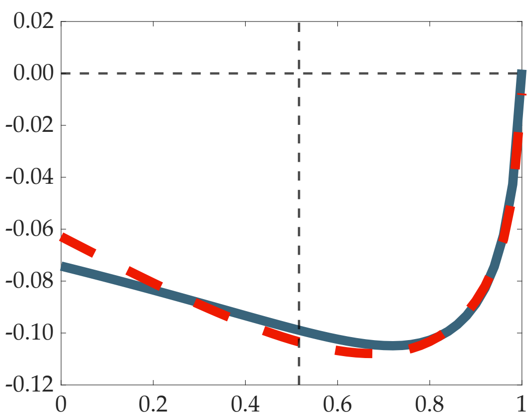
(b) Output



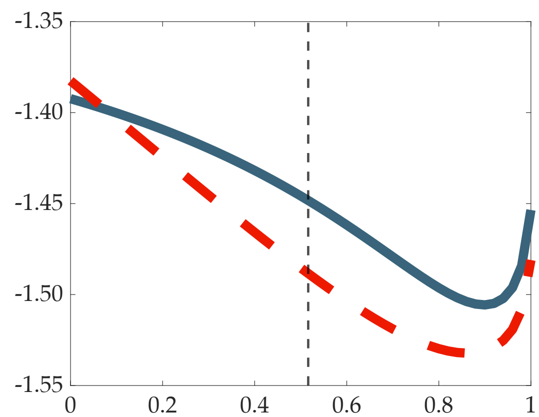
(c) Real interest rate



(d) Consumption



(e) Trade balance/GDP



(f) Real exchange rate

Notes: This figure compares the impact response of aggregate variables to a one-standard-deviation shock in commodity prices under different configurations for nominal rigidities. Baseline denotes the case with nominal rigidities in all sectors. Counterfactual denotes the case of flexible prices in other domestic sectors, but not imports. The horizontal axis corresponds to the degree of nominal rigidities in importable goods. Vertical axis in percentage deviations from steady state, except for trade balance/GDP (level deviations). Vertical dashed line marks the baseline degree of import rigidities.

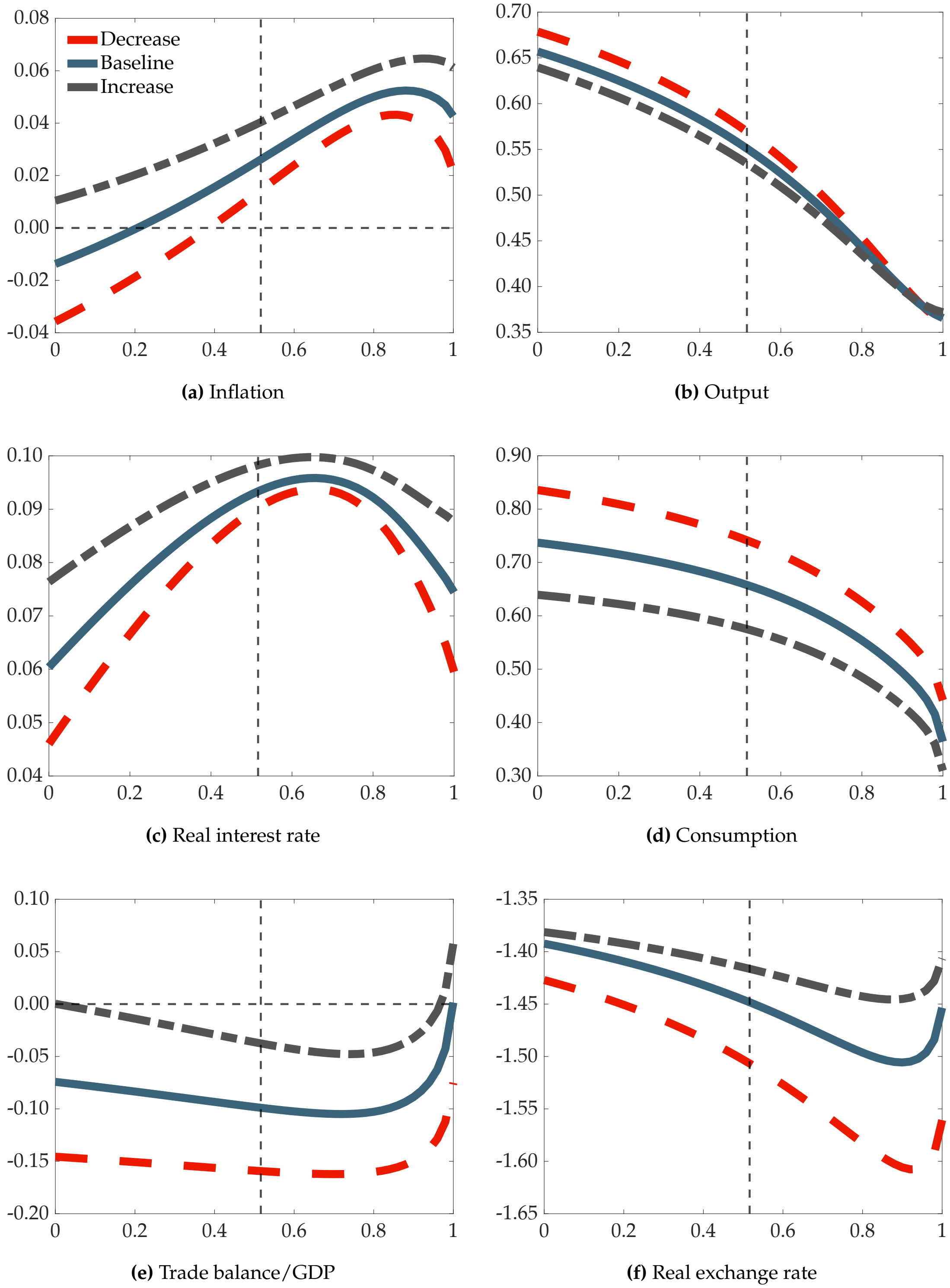
From our theoretical results in Section 3.3, we know that stronger usage of domestic materials increases the propagation of the shock by increasing the network multiplier and the Leontief inverse, which generates more inflation. However, this effect is dampened and eventually muted if the recomposition of labor and imports is strong enough.

Fig. 2 presents the results of this exercise. Consistent with the previous intuition, panel a shows that an economy with more intensive material usage experiences higher inflation after a commodity price shock. However, note that the intensity of material usage dramatically changes the threshold value of nominal rigidities of imports discussed earlier. In our baseline calibration, such a threshold is approximately 20 percent and doubles after a ten percent decrease in material usage. This reflects that in an economy with weaker production linkages, the import-relief channel is stronger. The opposite occurs when an economy is more interconnected: the wage-push channel prevails, and the economy never experiences deflation after the commodity price shock. These results can be summarized as follows: an increase in the use of materials strengthens network amplification unless compensated by reallocation away from labor/imports; this shifts the inflation sign-flip threshold to the right when imports are sticky.

Even though there are significant quantitative differences in the response of inflation, this is not the case for output in panel b. More connections generated a dampened response of output, but the differences are slight. This is consistent with the results presented in Romero (2025), which shows that more connected economies experience a lower response of real GDP after a commodity price shock. This result follows from the propagation of the shock and the assumption of perfect competition of the commodity sector. Because marginal costs increase, but the commodity price can not adjust, the commodity sector decreases its demand for inputs relative to a less connected economy. Combining the large differences in the inflation response with the nuanced variations in GDP, the real interest responds more strongly in more connected economies (panel c), which translates one-to-one into larger differences in consumption (panel d).

In this exercise, the differences in the trade balance and the real exchange rate are larger across scenarios. As shown in panel e, the stronger the increase in the usage of materials, the lower the trade balance deficit. This is because there are fewer imports by the productive sectors. Hence, the increase in exports dominates the overall response, and there is no need to generate such a significant real appreciation to compensate (panel f).

Figure 2. The Role of Production Linkages



Notes: This figure compares the impact response of aggregate variables to a one-standard-deviation commodity price shock under different configurations for material usage. Baseline denotes the case with nominal rigidities in all sectors. Counterfactual denotes the case of flexible prices in other domestic sectors, but not imports. The horizontal axis corresponds to the degree of nominal rigidities in importable goods. Vertical axis in percentage deviations from steady state, except for trade balance/GDP (level deviations). Vertical dashed line marks the baseline degree of import rigidities.

4.4 Extensions

Once the main results of the paper, regarding the interaction between nominal rigidities and production linkages, are established, this section provides further robustness and extensions.

Changes in complementarities of materials. Our baseline calibration follows Atalay (2017) and assumes that sectoral goods are complements in the bundle of materials ($\sigma_M = 0.1$). Fig. B.2 presents the impact response of the economy to the commodity price shock under alternative scenarios for the elasticity of substitution of materials. The main observation derived from this figure is that this elasticity is second order for the equilibrium of the economy, due to the similar responses in comparison with the Cobb-Douglas ($\sigma_M = 1$) and the complements case ($\sigma_M = 2$).

Changes in complementarities of inputs. As the response of the economy is governed by the strength of the wage-push channel relative to the imports-relief channel, the elasticity of substitution between labor, imports, and materials should play a significant role in the economy's response.

Fig. B.3 compares the impact response of the economy in the baseline case ($\sigma_Q = 1$) against scenarios with gross complementarities ($\sigma_Q = 0.1$) and gross substitution ($\sigma_Q = 2$). Interestingly, our results show that the threshold of import nominal rigidities shifts to the right when there are complementarities in production, suggesting that the import-cost relief channel dominates over a larger parameter space. This implies lower inflationary pressure, a less responsive monetary policy, and a stronger reaction in consumption. However, as the response of the trade balance dominates that of consumption, it leads to a significant increase in imports following the shock, a stronger appreciation, and a lower response of output.

Other driving forces. While the primary focus of this paper is the impact of commodity price shocks in an emerging economy, the transmission channels of production linkages and nominal rigidities could also be important in the context of other shocks. How do they interact when there are different driving forces at play?

To answer this question, we analyze the interaction of production linkages and nominal rigidities when the economy is subject to other driving forces relevant for a small open economy.[9] In particular, we study the responses under (i) aggregate technology shocks; (ii) domestic monetary policy shocks;

[9]See for example Neumeyer and Perri (2005), Uribe and Yue (2006), Fernandez and Gulan (2015), Kohn et al. (2020) and Garcia-Schmidt and Garcia-Cicco (2020).

(iii) shocks to the foreign interest rate premium; and (iv) foreign demand shocks. Except for the domestic monetary policy shock, we assume that all these shocks follow an AR(1) process in logs. In each case, the magnitude of the shock is one percent. This exercise aims not to provide a fully calibrated analysis but to illustrate the importance and implications of the main channels at play in the transmission of shocks. Appendix B.3 presents all results.

From these exercises, we get the following lesson. When the economy's driving forces directly impact marginal costs (such as aggregate TFP shocks or the commodity price shock by affecting the real exchange rate and the price of imported goods), the interaction between nominal rigidities, IO linkages, and complementarities in production distorts the response of the economy. On the contrary, when there is no direct effect on marginal costs for other forces (such as the foreign interest rate shock), the implications are through aggregate demand in general equilibrium. Therefore, in the latter cases, production linkages amplify or dampen the economy's response but do not distort how nominal rigidities affect the economy. Moreover, the degree of complementarities in production plays a more relevant role in the propagation of shocks.

5 Conclusion

In a commodity-exporting small open economy with production linkages, *where* prices are sticky, it is as important as *how much* they are sticky. In this paper, we demonstrate that a commodity price increase triggers a wage-push versus an import-cost relief tension, while production linkages propagate whichever force prevails. When import prices are rigid, the real exchange rate appreciates slowly, input costs do not decrease enough, and inflation rises, leading monetary policy to lean against activity and mimicking a positive demand shock. When import prices are flexible, the appreciation lowers marginal costs quickly, inflation falls, and activity expands, mimicking an expansionary supply shock. Stronger production linkages shift the threshold between these regimes and generally dampen the response of output.

The same logic extends to other driving forces that directly affect marginal costs, which interact most with nominal rigidities and production linkages. Policy analysis in small open economies should therefore measure and model the location of nominal rigidity – including import-price stickiness and pass-throughs – alongside network structure. Doing so sharpens monetary-policy guidance in the face of terms of trade volatility and helps interpret why identical external shocks can generate opposite inflationary outcomes across otherwise similar economies.

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A Theoretical Results

This appendix summarizes the proofs of the propositions presented in Section 3. We start with Lemma 1 which guarantees the existence of the Leontief inverse of the economy.

Lemma 1. *Considering* $\mathbf{H} = (\mathbf{I} - \mathbf{B})^{-1}$ *with* $\mathbf{B} = \operatorname{diag}(\widetilde{\boldsymbol{\kappa}})\operatorname{diag}(\boldsymbol{\mu})\boldsymbol{\Gamma}'$ *and recalling that* $\widetilde{\kappa}_j \in [0,1]$, $\mu_j \in [0,1]$, $\gamma_{ij} \geq 0$ *and* $\sum_i \gamma_{ij} = 1$ *for every* j, *then*

1. $\mathbf{B} \geq 0$ *elementwise.*

2. *The spectral radius of* $\mathbf{B}$, $\rho(\mathbf{B})$, *(the largest absolute value of eigenvalues of* $\mathbf{B}$*) satisfies* $\rho(\mathbf{B}) \leq ||\mathbf{B}||_\infty = \max_i \widetilde{\kappa}_i\mu_i < 1$. *Therefore,* $\mathbf{H}$ *exists and* $\mathbf{H} = \sum_{n=0}^{\infty}\mathbf{B}^n \geq 0$ *elementwise.*

Proof. The first part is immediate because each element in $\mathbf{B}$ is positive. For the second part, because $\boldsymbol{\Gamma}'$ has row sums equal to 1, the i-th row sum of $\mathbf{B}$ is:

$$\sum_j \mathbf{B}_{ij} = [\operatorname{diag}(\widetilde{\boldsymbol{\kappa}})\operatorname{diag}(\boldsymbol{\mu})]_{ii}\sum_j \boldsymbol{\Gamma}'_{ij} = \widetilde{\kappa}_i\mu_i,$$

where $[\mathbf{X}]_{ii}$ denotes the (i,i) element of $\mathbf{X}$. Thus, $||\mathbf{B}||_\infty = \max_i \widetilde{\kappa}_i\mu_i < 1$ whenever each sector uses some labor or imports ($\alpha_i + \nu_i > 0$ or equivalently $\mu_i < 1$). Hence $\rho(\mathbf{B}) \leq ||\mathbf{B}||_\infty < 1$, implying that $\mathbf{I} - \mathbf{B}$ is invertible and $\mathbf{H} = \sum_{n=0}^{\infty}\mathbf{B}^n \geq 0$. □

A.1 Proof of Proposition 1.

Proof. From previous definitions, note that only the vector $\mathbf{a}$ depends on $\widetilde{\kappa}_v$, but not the network multiplier $\mathbf{H}$. Therefore

$$\frac{\partial \pi_t}{\partial \widetilde{\kappa}_v} = \boldsymbol{\vartheta}'\mathbf{H}\left(\frac{\partial \mathbf{a}}{\partial \widetilde{\kappa}_v}\right)\widehat{p}^*_{1t} = \boldsymbol{\vartheta}'\mathbf{H}\big(\operatorname{diag}(\widetilde{\boldsymbol{\kappa}})\boldsymbol{\nu}\kappa_q\big)\widehat{p}^*_{1t} < 0,$$

holds because $\kappa_q < 0$. □

A.2 Proof of Proposition 2

Proof. Differentiating aggregate inflation with respect to $\widetilde{\kappa}_j$, we get:

$$\frac{\partial \pi_t}{\partial \widetilde{\kappa}_j} = \boldsymbol{\vartheta}'\left(\frac{\partial \mathbf{H}}{\partial \widetilde{\kappa}_j}\mathbf{a} + \mathbf{H}\frac{\partial \mathbf{a}}{\partial \widetilde{\kappa}_j}\right)\widehat{p}^*_{1t}.$$

The partial derivatives of each term read as:

$$\frac{\partial \mathbf{H}}{\partial \widetilde{\kappa}_j} = \mathbf{H}\Big(\frac{\partial \mathbf{B}}{\partial \widetilde{\kappa}_j}\Big)\mathbf{H} = \mathbf{H}\Big(\frac{\partial \operatorname{diag}(\widetilde{\boldsymbol{\kappa}})}{\partial \widetilde{\kappa}_j}\operatorname{diag}(\boldsymbol{\mu})\boldsymbol{\Gamma}'\Big)\mathbf{H} = \mathbf{H}\Big(\mathbf{E}_j \operatorname{diag}(\boldsymbol{\mu})\boldsymbol{\Gamma}'\Big)\mathbf{H}$$

$$\frac{\partial \mathbf{a}}{\partial \widetilde{\kappa}_j} = \mathbf{E}_j \mathbf{u},$$

where $\mathbf{E}_j$ is a selection matrix taking value one for element (j, j) and where $\frac{\partial \mathbf{a}}{\partial \widetilde{\kappa}_j}$ captures the direct impact of price flexibility while $\frac{\partial \mathbf{H}}{\partial \widetilde{\kappa}_j}$ captures the network-multiplier effect.

Nothing that this latter effect is always positive (by Lemma 1), if all the elements in $\mathbf{u}$ are nonnegative, the direct effect has a positive impact, and an increase in price flexibility in sector j is inflationary. □

A.3 Proof of Proposition 3

Proof. As shown in the main text, GDP reads as $\widehat{Y}_t = -\frac{\phi_\pi}{\sigma+\phi_y}\pi_t$. Therefore, the partial derivative with respect to each price flexibility term is the negative of the response of inflation, as indicated in Propositions 1 and 2. □

B Quantitative Results

B.1 Calibration

Table B.1 presents the model calibration. Figure Fig. B.1 compares the empirical and theoretical responses to a one standard deviation commodity price shock. From the figure, the model fairly replicates both the impact and the shape of the response of aggregate variables after a commodity price shock.

B.2 Additional Results for Commodity Price Shocks

This section presents additional results on the role of nominal rigidities in import prices and other model characteristics when the economy is subject to a commodity price shock.

Fig. B.2 compares the impact responses for different degrees of nominal rigidities in imports under different calibrations for the elasticity of substitution for the bundle of materials (ε_M). Fig. B.3 presents similar results but for different calibrations for the elasticity of substitution of inputs in the production function (ε_Q).

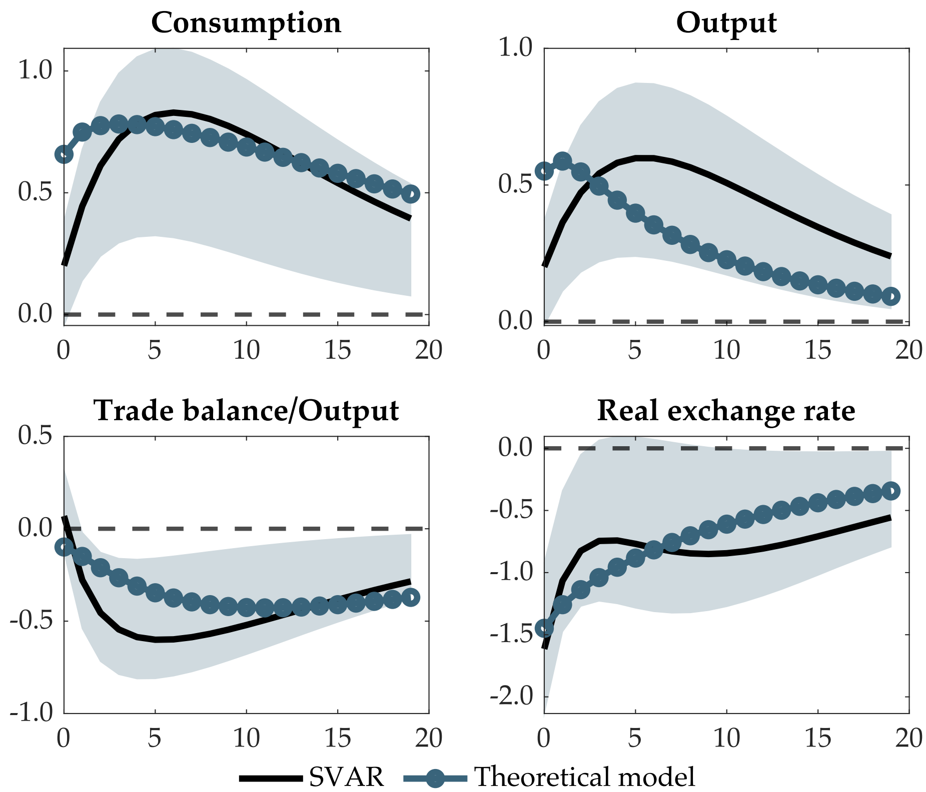
Our results show that the elasticity of substitution of inputs is crucial for the aggregate response of the economy to a commodity price shock, as it alters the relative importance of the wage-push

Table B.1. Calibration

Variable	Parameter	Value	Source/Target
Panel A: Household			
σ	Risk aversion	1	Standard value
φ	Frisch elasticity	1/0.54	Chetty (2011)
β	Discount factor	0.986	Garcia-Schmidt and Garcia-Cicco (2020)
ϱ	Elasticity of substitution (labor)	1	Standard value
ϑ_j	Sectoral consumption shares	$\{\vartheta_m = 0.4; \vartheta_s = 0.6\}$	IO (2017)
Panel B: Sectors			
ε	Elasticity of substitution (varieties)	10	Standard value
ε_Q	Elasticity of substitution (production)	1	Atalay (2017)
ε_M	Elasticity of substitution (materials)	0.1	Atalay (2017)
α_j	Labor share	$\{\alpha_c = 0.19; \alpha_m = 0.24; \alpha_s = 0.48\}$	IO (2017)
μ_j	Materials share	$\{\mu_c = 0.69; \mu_m = 0.60; \mu_s = 0.45\}$	IO (2017)
γ_{ij}	Production linkages		IO (2017)
θ_j	Price rigidities (Calvo)	$\{\theta_m = 0.46; \theta_s = 0.59\}$	Chilean microdata
ψ_j	Share in exports	$\{\psi_m = 0.83; \psi_s = 0.17\}$	IO (2017)
Panel C: Monetary policy and aggregates			
ϕ_π	Weight on inflation	1.5	Standard value
ϕ_y	Weight on GDP	0.125	Standard value
R^w	World interest rate	1.045	Garcia-Schmidt and Garcia-Cicco (2020)
$\bar{b}$	Interest rate premium	-5.624	TB/GDP = 0.08
ω^x	Foreign demand for exports	1	Normalization
η^*	Foreign demand for exports	1.4	Adolfson et al. (2007)
θ_v	Price rigidities (imports)	0.52	Campa and Goldberg (2005)
ϕ_b	Interest rate premium	0.009	SVAR calibration
ρ_1	Persistence commodity price	0.868	SVAR estimation
σ_1	Volatility of commodity price	0.014	SVAR estimation

Notes: This table presents the calibration of the model. See Section 4.1 for details.

Figure B.1. Impulse-response Function to a One Standard Deviation Commodity Price Shock



Notes: Each blue area corresponds to the 95% confidence intervals computed with bootstrap with 100,000 replications. The SVAR model includes a constant and a quadratic trend as exogenous controls. Horizontal axes in quarters. Vertical axes correspond to percentage deviations with respect to trend (for empirical model) and percentage deviations with respect to steady-state (for model-implied responses), except for trade balance/GDP (in deviations).

and import-cost relief channels. On the other hand, the elasticity of substitution governing the bundle of materials is second order.

B.3 Other Driving Forces

This section examines the interaction between nominal rigidities and other model characteristics when other variables drive the cycle of the small open economy. Tables B.2 to B.5 presents the results for shocks to (i) aggregate TFP, (ii) domestic monetary policy, (iii) the foreign interest rate premium, and (iv) foreign GDP. We summarize our results in turn.

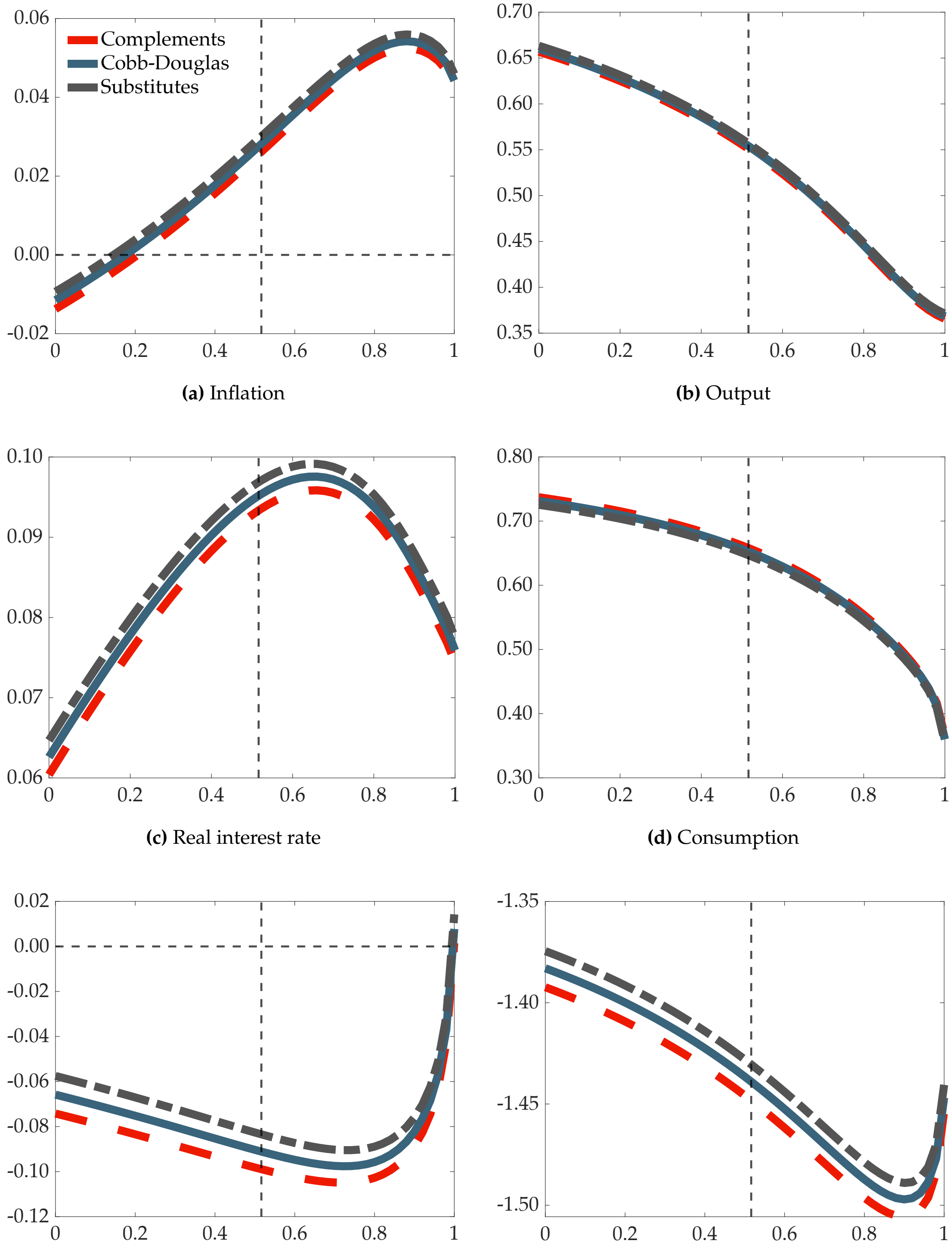
Aggregate TFP. Conceptually, this shock affects all sectors in the economy by reducing their marginal cost of production and prices. As Table B.2 shows, inflation decreases in response to the shock, accompanied by a significant decline in the real interest rate, which stimulates aggregate consumption and total output. The decrease in prices associated with the shock generates an appreciation of the exchange rate and a trade balance surplus.

When the economy is more connected through the use of materials, the response of inflation is smaller due to a stronger response of the real exchange rate. This effect implies a milder response of monetary policy, accompanied by an increase in output and consumption. These results are in line with previous literature (see, among others, Acemoglu et al., 2016) where a decrease in production linkages dampens the response to the shock. Moreover, these forces are stronger when import prices are flexible and when there are complementarities in production inputs.

Domestic monetary policy. An increase in the domestic monetary policy rate directly increments the real rate, which postpones consumption and decreases output, as shown in Table B.3. The implication is a lower demand for sectoral goods and a decrease in aggregate inflation. Note that the combination of a higher interest rate and a reduction in inflation dominates, resulting in an appreciating real exchange rate upon impact.

Although the degree of material usage does not generate significant changes in the economy's response, the complementarity of inputs does. In particular, after the monetary policy shock, in an economy with higher complementarities, we observe lower responses in quantities (output and consumption) and a stronger response to inflation, especially when import prices are flexible.

Figure B.2. Changes in Complementarities of Materials



(a) Inflation

(b) Output

(c) Real interest rate

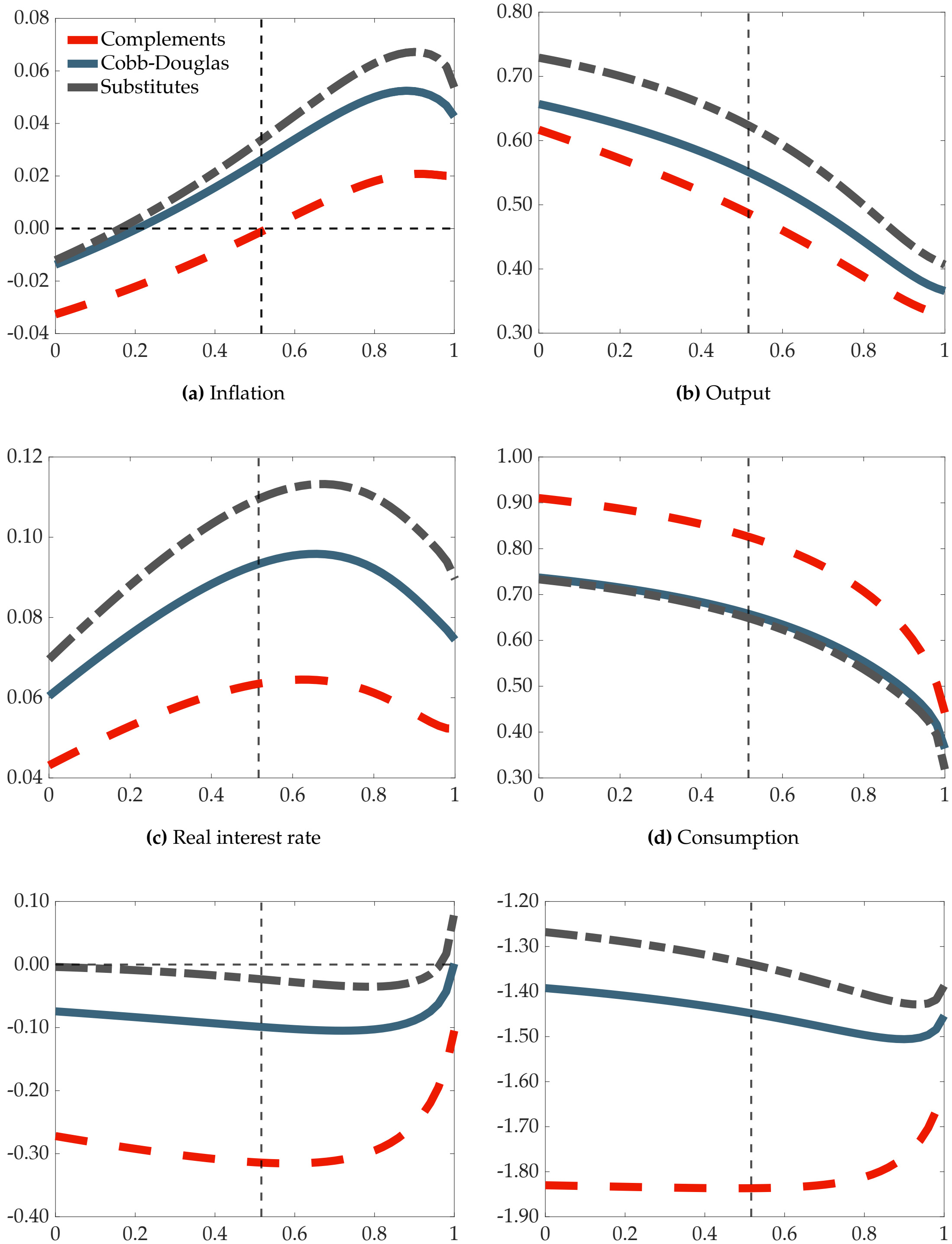
(d) Consumption

(e) Trade balance/GDP

(f) Real exchange rate

Notes: This figure compares the impact response of aggregate variables to a one-standard-deviation commodity price shock under different configurations for complementarities in material usage. Baseline denotes the baseline calibration. Complements assumes $\sigma_M = 0.1$. Cobb-Douglas assumes $\sigma_M = 1.0$. Substitutes assumes $\sigma_M = 2.0$. Vertical axis in percentage deviations from steady state, except for trade balance/GDP (level deviations). Vertical dashed line marks the baseline degree of import rigidities.

Figure B.3. Changes in Complementarities of Inputs



(a) Inflation

(b) Output

(c) Real interest rate

(d) Consumption

(e) Trade balance/GDP

(f) Real exchange rate

Notes: This figure compares the impact response of aggregate variables to a one-standard-deviation commodity price shock under different configurations for complementarities in input usage. Baseline denotes the baseline calibration. Complements assumes $\sigma_Q = 0.1$. Cobb-Douglas assumes $\sigma_Q = 1.0$. Substitutes assumes $\sigma_Q = 2.0$. Vertical axis in percentage deviations from steady state, except for trade balance/GDP (level deviations). Vertical dashed line marks the baseline degree of import rigidities.

Foreign interest rate. A positive foreign interest rate shock (either by an increase in the world interest rate or the risk premium of the small open economy) implies an increase in the domestic rate, as can be observed by approximating and combining the Euler equations of the household. This higher interest rate postpones consumption and increases savings today, resulting in a trade balance surplus. At the same time, the larger expansion in the trade balance depreciates the real exchange rate, which combines with a positive response to inflation (Table B.4).

When there is an increase in the use of materials, this implies a lower demand for imports and a trade balance surplus. This is due to a stronger response in output and a slight increase in inflation. The opposite happens with complementarities in production inputs.

Foreign GDP. An increase in foreign GDP translates into an increase in the foreign demand for domestic goods. As shown in Table B.5, this shock increases the value of total exports in the economy, so we observe a trade balance surplus and an exchange rate appreciation. Even though the consumption response is nil, the increase in foreign demand necessitates a higher production of domestic goods, which in turn leads to inflation and a corresponding response from monetary policy. The combined response implies an immediate increase in GDP. All these responses are dampened by the stronger usage of materials, but are enhanced by complementarities in production.

Table B.2. Impact Responses to a TFP Shock

	Inflation	Output	Real Interest Rate	Consumption	Trade Balance	Real Exchange Rate
Panel A: Baseline calibration						
	-0.490	2.896	-0.185	1.599	1.221	-0.011
Panel B: 10% increase in the usage of materials						
Baseline rigidities	-0.477	3.510	-0.096	1.722	1.688	-0.130
Imports flexible	-0.485	3.544	-0.101	1.751	1.694	-0.121
Panel C: Complementarities in inputs						
Baseline rigidities	-0.581	2.636	-0.310	1.761	0.822	-0.039
Imports flexible	-0.587	2.665	-0.309	1.795	0.818	-0.045

Notes: This table compares the impact responses of the economy after an aggregate productivity shock. Panel A presents results using the calibration presented in Table B.1. Panel B increases the intensity of usage in materials (μ_j) by 10%. Panel C changes the elasticity of substitution of production to $\sigma_Q = 0.1$. All numbers in percentage deviation from steady state, except for trade balance (level deviations).

Table B.3. Impact Responses to a Monetary Policy Shock

	Inflation	Output	Real Interest Rate	Consumption	Trade Balance	Real Exchange Rate
Panel A: Baseline calibration						
	-0.552	-1.205	0.333	-0.996	-0.190	-0.708
Panel B: 10% increase in the usage of materials						
Baseline rigidities	-0.534	-1.548	0.312	-1.126	-0.384	-0.645
Imports flexible	-0.559	-1.456	0.297	-1.065	-0.355	-0.616
Panel C: Complementarities in inputs						
Baseline rigidities	-0.582	-1.032	0.321	-0.917	-0.105	-0.646
Imports flexible	-0.603	-0.945	0.309	-0.857	-0.080	-0.643

Notes: This table compares the impact responses of the economy after a monetary policy shock. Panel A presents results using the calibration presented in Table B.1. Panel B increases the intensity of usage in materials (μ_j) by 10%. Panel C changes the elasticity of substitution of production to $\sigma_Q = 0.1$. All numbers in percentage deviation from steady state, except for trade balance (level deviations).

Table B.4. Impact Responses to a Foreign Interest Rate Premium

	Inflation	Output	Real Interest Rate	Consumption	Trade Balance	Real Exchange Rate
Panel A: Baseline calibration						
	0.443	0.564	0.444	-2.060	2.396	1.984
Panel B: 10% increase in the usage of materials						
Baseline rigidities	0.465	0.939	0.517	-1.832	2.539	1.511
Imports flexible	0.505	0.795	0.547	-1.926	2.490	1.466
Panel C: Complementarities in inputs						
Baseline rigidities	0.433	0.335	0.398	-2.465	2.549	3.015
Imports flexible	0.492	0.088	0.439	-2.626	2.467	3.005

Notes: This table compares the impact responses of the economy after a shock to the interest rate premium. Panel A presents results using the calibration presented in Table B.1. Panel B increases the intensity of usage in materials (μ_j) by 10%. Panel C changes the elasticity of substitution of production to $\sigma_Q = 0.1$. All numbers in percentage deviation from steady state, except for trade balance (level deviations).

Table B.5. Impact Responses to a Foreign GDP Shock

	Inflation	Output	Real Interest Rate	Consumption	Trade Balance	Real Exchange Rate
Panel A: Baseline calibration						
	0.012	0.039	0.018	-0.001	0.037	-0.050
Panel B: 10% increase in the usage of materials						
Baseline rigidities	0.008	0.035	0.013	0.000	0.032	-0.035
Imports flexible	0.008	0.037	0.012	0.001	0.033	-0.034
Panel C: Complementarities in inputs						
Baseline rigidities	0.016	0.043	0.023	0.002	0.038	-0.086
Imports flexible	0.015	0.048	0.022	0.004	0.040	-0.085

Notes: This table compares the impact responses of the economy after a shock to foreign GDP. Panel A presents results using the calibration presented in Table B.1. Panel B increases the intensity of usage in materials (μ_j) by 10%. Panel C changes the elasticity of substitution of production to $\sigma_Q = 0.1$. All numbers in percentage deviation from steady state, except for trade balance (level deviations).